Zero and Negative Exponents

Exercises

Write each expression as an integer, a simple fraction, or an expression that contains only positive exponents. Simplify.

- **1.** 2.3⁰ **1 2.** 10^{-4} $\frac{1}{10,000}$ **3.** $2a^{-5} \frac{2}{a^5}$ **4.** 113.7⁰ **1** 6. $\frac{3^{-3}}{p}$ $\frac{1}{270}$ 5. 19^{-1} $\frac{1}{19}$ 8. $\left(-\frac{7}{8}\right)^{-2}$ $\frac{64}{49}$ 7. $(7q)^{-1} \frac{1}{7q}$
- **9.** 1.8*c*⁰ **1.8 10.** $(-9.7)^0$ **1**

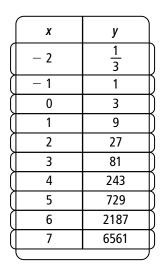
Write each expression so that it contains only positive exponents. Simplify.

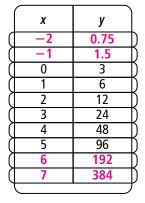
- **11.** -6^{-3} $-\frac{1}{216}$ **12.** $-2rs^{-5} - \frac{2r}{c^5}$
- 14. $\left(\frac{5a}{3b}\right)^{-2} \frac{9b^2}{25a^2}$ **13.** $7x^{-8}y^0 \frac{7}{x^8}$
- 16. $\frac{2^{-3}}{m^0 n^{-1}} \frac{n}{8}$ **15.** $(-8v)^{-2}w^3 \frac{w^3}{64v^2}$
- 18. $\frac{-3^{-3}}{uv^{-2}} \frac{v^2}{27u}$ **17.** $(3xy)^0 z z$

Reteaching (continued)

Exponential Functions

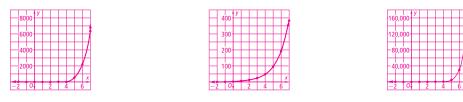
1. For each of the tables on the previous page, extend them two units in each direction. Use the common difference in the *x*-values and the common ratio in the *y*-values to do the extension. The first table is done for you.





1	x	у	
(-2	0.08	D
(-1	0.4	D
J	0	2	D
(1	10	D
(2	50	D
(3	250	D
(4	1250	D
(5 6	6250	D
(6	31,250	D
J	7	156,250	D
			ſ

2. Plot the points in each of your extended tables on separate coordinate grids. Connect the points with a smooth curve. The domain of each function is all real numbers and that the range is all positive real numbers. Explain why there are negative values for *x* but not for *y*.



The domain is all real numbers because x can have any value, but the range is all positive real numbers because a > 0 and $b^x > 0$, so $a \cdot b^x$ will always be > 0.

3. For each of the tables, identify the starting value *a* and the common ratio *b*. For the first table, *a* is 1 and *b* is 3. Next, write the exponential function that describes each table. The function for the first table is $f(x) = 1 \cdot 3^x$. Check if your function is correct by substituting in *x*-values and seeing if the function produces values for *y* that match the values in the table.

Table 1: 1; 3; $1 \cdot 3^{x}$; Table 2: 3; 2; $3 \cdot 2^{x}$; Table 3: 2; 5; $2 \cdot 5^{x}$;

Reteaching (continued)

Comparing Linear and Exponential Functions

Problem

What is the average rate of change for the function $f(x) = 2 \cdot 2^x$ over the intervals $-2 \le x \le 0$, $0 \le x \le 2$, and $2 \le x \le 4$? Describe what you observe.

Step 1 Make a table of values.

							<u> </u>	2
X	-2	-1	0	1	2	3	4	D
f (x)	0.5	1	2	4	8	16	32	IJ
							λ	Τ

Step 2 Find the average rates of change over the intervals $-2 \le x \le 0$, $0 \le x \le 2$, and $2 \le x \le 4$.

$$\frac{f(b) - f(a)}{b - a} = \frac{f(0) - f(-2)}{0 - (-2)} = \frac{2 - 0.5}{2} = 0.75$$
$$\frac{f(b) - f(a)}{b - a} = \frac{f(2) - f(0)}{2 - 0} = \frac{8 - 2}{2} = 3$$
$$\frac{f(b) - f(a)}{b - a} = \frac{f(4) - f(2)}{4 - 2} = \frac{32 - 8}{2} = 12$$

The average rate of change is different over each interval. It increases with each interval. So the rate of change is increasing.

Exercises

1. Does the table represent a *linear function* or an *exponential function*? Explain.

X	0	1	2	3	D
y	15	20	25	30	V

The difference between each x-value is 1 and the difference between each y-value is 5. The table represents a linear function because there is a common difference between x-values and a common difference between y-values.

- 2. Suppose that the fish population of a lake decreases by half each week. Can you model the situation with a linear function or an exponential function? Explain. The difference between each x-value is 1 and the ratio between each y-value is 0.5. You can model the situation with an exponential function.
- **3.** Find the average rate of change for the function $y = 2.5 \cdot 2^x$ over the intervals $1 \le x \le 3$, $3 \le x \le 5$, and $5 \le x \le 7$. Describe what you observe.

$$\frac{f(3) - f(1)}{3 - 1} = 7.5 \qquad \qquad \frac{f(5) - f(3)}{5 - 3} = 30 \qquad \qquad \frac{f(7) - f(5)}{7 - 5} = 120$$

The average rate of change is different over each interval. It increases with each interval. So the rate of change is increasing.

Reteaching

Exponential Growth and Decay

Exponential functions can model the growth or decay of an initial amount.

The basic exponential function is $y = a \cdot b^x$ where

a represents the initial amount

b represents the growth (or decay) factor. The growth factor equals 100% plus the percent rate of change. The decay factor equals 100% minus the percent rate of decay.

x represents the number of times the growth or decay factor is applied.

y represents the result of applying the growth or decay factor *x* times.

Problem

A gym currently has 2000 members. It expects to grow 12% per year. How many members will it have in 6 years?

There are 2000 members to start, so a = 2000. The growth per year is 12%, or 0.12, so b = 1 + 0.12 or 1.12. The desired time period is 6 years, so x = 6. The function is $y = 2000 \cdot 1.12^{x}$.

When x = 6, $y = 2000 \cdot 1.12^6 \approx 3948$. So, the gym will have about 3948 members in 6 years.

In Exercises 1–3, identify *a*, *b*, and *x*. Then use them to write the exponential function that models each situation. Finally use the function to answer the question.

1. When a new baby is born to the Johnsons, the family decides to invest \$5000 in an account that earns 7% interest as a way to start the baby's college fund. If they do not touch that investment for 18 years, how much will there be in the college fund?

```
a = 5000, b = 1.07, x = 18; y = 5000(1.07)^{18}; $16,899.66
```

2. The local animal rescue league is trying to reduce the number of stray dogs in the county. They estimate that there are currently 400 stray dogs and that through their efforts they can place about 8% of the animals each month. How many stray dogs will remain in the county 12 months after the animal control effort has started?

 $a = 400, b = 0.92, x = 12; y = 400(0.92)^{12}; 147$ stray dogs

3. A basket of groceries costs \$96.50. Assuming an inflation rate of 1.8% per year, how much will that same basket of groceries cost in 20 years?

 $a = 96.5, b = 1.018, x = 20; y = 96.5(1.018)^{20};$ \$137.87

Name	Class	Date

Exponential Growth and Decay

While it is usually fairly straightforward to determine a, the initial value, you often need to read the problem carefully to make sure that you are correctly identifying b and x. This is especially true when considering situations where the given growth rate is applied in intervals that are not the same as the given value of x.

Problem

You invest \$1000 in an account that pays 8% interest. If nothing changes, a = 1000, b = 1.08, and x = 3. The function is $y = 1000 \cdot 1.08^3$. How much money will you have left after 3 years?

In this case, the interest is *compounded annually*, meaning it is added to your account at the end of each year. But what happens if the interest is *compounded quarterly*, meaning it is added to your account at the end of each quarter of a year?

There will be 12 compounding periods in the 3-year period.

In each compounding period you add the appropriate fraction of the total annual interest, in this case one-fourth of the interest.

The new function is $y = 1000 \cdot \left(1 + \frac{0.08}{4}\right)^{12}$, given that *a* is the initial investment,

b is the amount of growth for each compounding period, and *x* is the number of compounding periods.

Compounded annually, the value of the investment will be \$1259.71 after 3 years. Compounded quarterly, the value will be \$1268.24.

In Exercises 4 and 5, identify *a*, *b*, and *x*. Then write the exponential function that models the situation. Finally, use the function to answer the question.

- 4. You invest \$2000 in an investment that earns 6% interest, compounded quarterly. How much will the investment be worth after 5 years?
 a = 2000, b = 1.015, x = 20; y = 2000(1.015)²⁰; \$2693.71
- **5.** You invest \$3000 in an investment that earns 5% interest, compounded monthly. How much will the investment be worth after 8 years?
- a = 3000, b = 1.004167, x = 96; y = 3000(1.004167)⁹⁶; \$4471.89
 6. The formula that financial managers and accountants use to determine the value of investments that are subject to compounding interest is

 $A = P\left(1 + \frac{r}{n}\right)^{nt}$ where *A* is the final balance, *P* is the initial deposit, *r* is the annual interest rate, *n* is the number of times the interest is compounded per year and *t* is the number of years. Redo Exercises 4 and 5 using this formula.

Redo 4: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2000\left(1 + \frac{0.06}{4}\right)^{4\cdot5} = 2000(1.015)^{20} = 2693.71 Redo 5: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 3000\left(1 + \frac{0.05}{12}\right)^{12\cdot8} = 3000(1.004167)^{96} = 4471.89

_____ Class _____ Date _____

Reteaching (continued)

Solving Exponential Equations

Exercises

Solve each exponential equation.

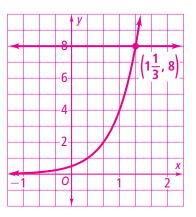
1.
$$5^x = 125$$
 x = **3 2.** $5^x = \frac{1}{125}$ **x** = -**3**

3.
$$\frac{1}{36} = 6^{x+5}$$
 $x = -7$ **4.** $2^{3x} = \frac{1}{64}$ $x = -2$

5. Find the solution of the equation $8 = 2^{3x-1}$ using both methods. **a.** Solve algebraically. Explain each step.

 $2^3 = 2^{3x-1}$ Rewrite 8 as a power with base 2. 3 = 3x - 1 Set the exponents equal to each other. $x = \frac{4}{3} = 1\frac{1}{3}$ Solve for *x*.

b. Solve graphically. Justify your solution.



The graphs intersect at the point $\left(1\frac{1}{3}, 8\right)$, so the solution is $x = 1\frac{1}{3}$.

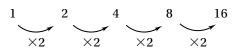
Reteaching

Geometric Sequences

A geometric sequence is a sequence in which the ratio between consecutive terms is constant. This ratio is called the common ratio. Every geometric sequence has a starting value and a common ratio.

Problem

Is the following a geometric sequence? 1, 2, 4, 8, 16, ...



The pattern is to multiply each term by 2 to get the next term. Because there is a common ratio r = 2, the sequence is geometric.

Determine whether the sequence is a geometric sequence. Explain.

- **1.** 5, 10, 20, 40, . . . yes; the sequence has a common ratio of 2
- **3.** 1, 4, 9, 16, . . . no; the sequence has no common ratio
- **5.** 25, 20, 15, 10, . . . no; the sequence has a common difference
- **2.** 3, 9, 27, 81, . . . yes; the sequence has a
- common ratio of 3 **4.** 24, 20, 16, 12, . . . no; the sequence has a common difference
- **6.** -48, 96, -192, 384, ...

ves; the sequence has a common ratio of -2

Any geometric sequence can be defined by both an explicit and a recursive definition. The recursive definition is useful for finding the next term in the sequence.

$$a_1 = a; a_n = a_{n-1} \cdot r$$

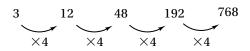
In this formula,

- *a*₁ represents the first term
- a_n represents the *n*th term
- *n* represents the term number
- *r* represents the common ratio
- a_{n-1} represents the term immediately before the *n*th term

Geometric Sequences

Problem

What is a recursive formula for the geometric sequence 3, 12, 48, 192, 768, ... ?



$a_1 = a; a_n = a_{n-1} \cdot r$	Use the recursive definition for a geometric sequence.
$a_1 = 3; a_n = a_{n-1} \cdot 4$	Replace <i>a</i> with 3 and <i>r</i> with 4.

Write the recursive formula for each geometric sequence.

7. 5, 25, 125, 625,	8. -2, 6, -18, 54,
$a_1 = 5; a_n = a_{n-1} \cdot 5$	$a_1 = -2; a_n = a_{n-1} \cdot (-3)$
9. 96, 72, 54, 40.5,	10. 10, -10, 10, -10,
$a_1 = 96; a_n = a_{n-1} \cdot \frac{3}{4}$	$a_1 = 10; a_n = a_{n-1} \cdot (-1)$

An explicit formula is more convenient when finding the *n*th term.

Problem

What is an explicit formula for the geometric sequence $-2, 2, -2, 2, \ldots$?

$a_n = a_1 \cdot r^{n-1}$	Use the explicit definition for a geometric sequence.
$a_n = -2 \cdot (-1)^{n-1}$	Replace a_1 with -2 and r with -1 .

Write the explicit formula for each geometric sequence.

11. -3, 3, -3, 3,	12. 1, 0.5, 0.25, 0.125,
$a_n = -3 \cdot (-1)^{n-1}$	$a_n = 1 \cdot (0.5)^{n-1}$
13. 200, 100, 50, 25,	14. 6, 12, 24, 48,
$a_n = 200 \cdot (0.5)^{n-1}$	$a_n = 6 \cdot 2^{n-1}$

Reteaching

Combining Functions

Adding and Subtracting Functions

You can add and subtract with functions just as you do with numbers or expressions.

Addition (f + g)(x) = f(x) + g(x)Subtraction (f - g)(x) = f(x) - g(x)

Problem

Find the sum if f(x) = 2x + 9 and g(x) = -5x + 6. What is (f - g)(8)?

Step 1 Find
$$(f - g)(x)$$
.
 $(f - g)(x) = f(x) - g(x)$
 $= (2x + 9) - (-5x + 6)$
 $= (2x + 5x) + (9 - 6)$
 $= 7x + 3$
Step 2 Evaluate $(f - g)(8)$.
 $(f - g)(8) = 7(8) + 3$
 $= 59$

Find the sum or difference if f(x) = x + 10 and g(x) = 2x - 8.

1. $(f+g)(x)$	2. $(f+g)(9)$	3. $(f-g)(x)$	4. $(f-g)(9)$
(f + g)(x) = 3x + 2	(f + g)(9) = 29	(f-g)(x)=-x+18	(f - g)(9) = 9

Problem

A band's concert earnings are given by the equations below, where x is the number of people in attendance. What function gives the band's total earnings? How much can the band expect to earn if 150 people attend the concert?

Merchandise sales: M(x) = 1.4xTicket sales: T(x) = 8x - 250**Step 1** Find (M + T)(x). **Step 2** Evaluate (M + T)(150). (M+T)(x) = M(x) + T(x)(M + T)(150) = 9.4(150) - 250=(1.4x)+(8x-250)= 1160=(1.4x+8x)-250= 9.4x - 250

The band can expect to earn \$1160 if 150 people attend the concert.

5. The functions below give the number of car sales for two dealerships *x* years after opening.

What is a function giving the difference in car sales between Dealership A and Dealership B? How many more cars is Dealership A expected to sell after 5 years?

Dealership A: A(x) = 35x + 650Dealership B: B(x) = 20x + 425(A - B)(x) = 15x + 225; (A - B)(5) = 300

Combining Functions

Multiplying and Dividing Functions

You can multiply and divide with functions just as you do with numbers or expressions.

Multiplication
$$(f \cdot g)(x) = f(x) \cdot g(x)$$
 Division $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ and $g(x) \neq 0$

Problem

Find the product if $f(x) = 5 \cdot 3^x$ and $g(x) = 2 \cdot 3^x$. What is $(f \cdot g)(2)$?

Step 1 Find $(f \cdot g)(x)$.Step 2 Evaluate $(f \cdot g)(2)$. $(f \cdot g)(x) = f(x) \cdot g(x)$ $(f \cdot g)(2) = 10 \cdot 3^{2(2)}$ $= (5 \cdot 3^x) \cdot (2 \cdot 3^x)$ $= 10 \cdot 3^4$ $= (5 \cdot 2) \cdot (3^x \cdot 3^x)$ = 810 $= 10 \cdot 3^{2x}$ = 810

Find the product or quotient if $f(x) = 3^x + 9$ and g(x) = 9x.

6.
$$(f \cdot g)(x)$$

7. $(f \cdot g)(2)$
8. $(\frac{f}{g})(x)$
9. $(\frac{f}{g})(2)$
($f \cdot g)(x) = 9x \cdot 3^{x} + 81x$
($f \cdot g)(2) = 324$
($\frac{f}{g})(x) = \frac{3^{x}}{9x} + \frac{1}{x}$
($\frac{f}{g})(2) = 1$
Problem

The number of orders a new restaurant expects to receive x days after opening is O(x) = 10x + 25. Of those, D(x) = 2x + 3 are expected to include dessert. Write a function P(x) giving the percentage of orders that include dessert. What percentage of orders is expected to include dessert after 10 days?

Step 1 Plan.Step 2 Find P(x).Step 3 Evaluate P(10).Set up a function P(x) by
multiplying the ratio of
dessert orders to food orders
by 100. Then solve the
equation for x = 10 days. $P(x) = \left(\frac{D}{O}\right)(x) \cdot 100$
 $= \frac{2x + 3}{10x + 25} \cdot 100$ $P(10) = \frac{2(10) + 3}{10(10) + 25} \cdot 100$
 $= \frac{23}{125} \cdot 100$
= 18.4

The restaurant can expect 18.4% of orders to include dessert.

10. The population of an endangered animal in *x* years is modeled by $f(x) = 120 \cdot 0.8^x$. The function g(x) = 1.7 represents the factor increase that a protection agency uses to predict the change in population due to new protections. Write a function giving the population after *x* years. What is the predicted population after 3 years?

 $(f \cdot g)(x) = 204 \cdot 0.8^{x}; (f \cdot g)(3) \approx 104$

Reteaching

Simplifying Radicals

You can remove perfect-square factors from a radicand.

Problem

What is the simplified form of $\sqrt{80n^5}$?

In the radic and, factor the coefficient and the variable separately into perfect square factors, and then simplify. Factor 80 and n^5 completely and then find paired factors.

Solve $80 = 8 \cdot 10 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$ $= (2 \cdot 2)(2 \cdot 2) \cdot 5 = (2 \cdot 2)^2 \cdot 5$ $\sqrt{80} = \sqrt{4^2 \cdot 5} = \sqrt{4^2} \cdot \sqrt{5}$ $= 4 \cdot \sqrt{5} = 4\sqrt{5}$ $n^5 = n \cdot n \cdot n \cdot n \cdot n$ $= (n \cdot n) \cdot (n \cdot n) \cdot n = (n \cdot n)^2 \cdot n$ $\sqrt{n^5} = \sqrt{(n \cdot n)^2} \cdot \sqrt{n}$ $= n^2 \cdot \sqrt{n} = n^2 \sqrt{n}$ $\sqrt{80n^5} = 4 \cdot n^2 \sqrt{5 \cdot n} = 4n^2 \sqrt{5n}$ Check $\sqrt{80n^5} \stackrel{?}{=} 4n^2 \sqrt{5n}$

 $\sqrt{80n^5} \stackrel{?}{=} 4n^2\sqrt{5n}$ $\frac{\sqrt{80n^5}}{\sqrt{5n}} \stackrel{?}{=} \frac{4n^2\sqrt{5n}}{\sqrt{5n}}$ $\sqrt{16n^4} \stackrel{?}{=} 4n^2$ $4n^2 = 4n^2 \checkmark$

Factor 80 completely. Find pairs of factors. Use the rule $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$. The square root of a number squared is the number: $\sqrt{a^2} = a$. Factor n^5 completely. Find pairs of factors. Separate the factors. Remove the perfect square. Combine your answers. Check your solution. Divide both sides by $\sqrt{5n}$.

Solution: The simplified form of $\sqrt{80n^5}$ is $4n^2\sqrt{5n}$.

Exercises

Simplify each radical expression.

1. $\sqrt{100n^3}$ 10 $n\sqrt{n}$	2. $\sqrt{120b^4}$ 2 $b^2\sqrt{30}$	3. $\sqrt{66t^5} t^2 \sqrt{66t}$
4. $\sqrt{32x}$ 4 $\sqrt{2x}$	5. $\sqrt{525c^7}$ 5 $c^3\sqrt{21c}$	6. $\sqrt{86t^2} t\sqrt{86}$
7. $\sqrt{50g^3}$ 5g $\sqrt{2g}$	8. $\sqrt{54h^6}$ $3h^3\sqrt{6}$	9. $\sqrt{35y} \sqrt{35y}$

Simplifying Radicals

Problem

What is the simplified form of $\sqrt{\frac{27t^4}{48t^2}}$?

Begin by cancelling the common factors in the numerator and denominator. Simplify the numerator and denominator separately when the denominator is a perfect square. Remember that the radical is not simplified if there is a radical in the denominator. Multiply to remove the radical from the denominator.

Solve

$$= \sqrt{\frac{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel$$

 $\sqrt{\frac{27t^3}{48t^4}} = \sqrt{\frac{3\cdot 3\cdot 3\cdot t\cdot t\cdot t}{3\cdot 4\cdot 4\cdot t\cdot t\cdot t\cdot t\cdot t}}$

$$=\frac{3}{4\sqrt{t}}$$

$$=\frac{3}{4\sqrt{t}}\frac{(\sqrt{t})}{(\sqrt{t})}$$

$$=\frac{3\sqrt{t}}{4\sqrt{t\cdot t}}=\frac{3\sqrt{t}}{4\sqrt{t^2}}=\frac{3\sqrt{t}}{4t}$$

Factor the numerator and denominator completely.

Cancel the common factors.

Find pairs of factors. These are the perfectsquare factors.

Simplify the numerator and denominator separately to remove the perfect-square factors. $\sqrt{3^2} = 3$ and $\sqrt{4^2t} = 4\sqrt{t}$

Multiply the numerator and denominator by \sqrt{t} to remove \sqrt{t} from the denominator.

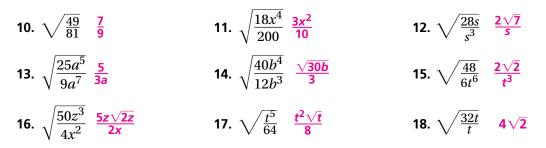
Remove the perfect-square factor from the denominator.

Solution: The simplified form of $\sqrt{\frac{27t^3}{48t^4}}$ is $\frac{3\sqrt{t}}{4t}$.

=

Exercises

Simplify each radical expression.



Radical and Piecewise Functions

Problem

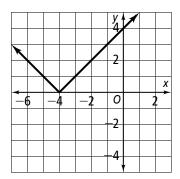
Graph the piecewise function $f(x) = \begin{cases} x+4, \text{ for } x \ge -4 \\ -x-4, \text{ for } x < -4 \end{cases}$.

Step 1 Make two tables.

x	f(x)=x+4
-4	-4 + 4 = 0
-3	-3 + 4 = 1
-2	-2 + 4 = 2
-1	-1 + 4 = 3

x	f(x)=-x-4
-5	-(-5) - 4 = 1
-6	-(-6) - 4 = 2
-7	-(-7) - 4 = 3
-8	-(-8) - 4 = 4

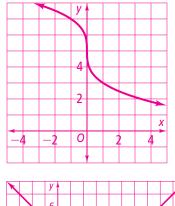
Step 2 Plot the points on a graph.

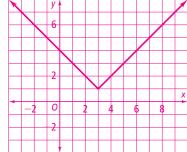


Exercises

Graph the function y = - ³√8x + 5. What are the domain and range of the function? The *x*-values -1, 0, 1, may help you start a table.
 domain: all real numbers range: all real numbers

2. Graph the piecewise function
$$f(x) = \begin{cases} x - 2, \text{ for } x \ge 3 \\ -x + 4, \text{ for } x < 3 \end{cases}$$





Copyright © by Pearson Education, Inc., or its affiliates. All Rights Reserved.