Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Simplifying Rational Expressions

Exercises

Simplify each expression. State any excluded values.

1. $\frac{a^4}{a}$ $a^3, a \neq 0$

2. $\frac{4x^3}{16x^2}$ $\frac{x}{4}; x \neq 0$

3. $\frac{cd^2}{3c^2d}$ $\frac{d}{3c}; d \neq 0, c \neq 0$

4. $\frac{lm}{l^2m^2n}$ $\frac{1}{lmn}; l \neq 0, m \neq 0, n \neq 0$

5. $\frac{64y}{16y^2x}$ $\frac{4}{xy}; x \neq 0, y \neq 0$

6. $\frac{2x^2 - 4x}{x}$ $2x - 4; x \neq 0$

7. $\frac{5x^3 - 15x^2}{x - 3}$ $5x^2; x \neq 3$

8. $\frac{x^2 + 5x + 6}{x + 3}$ $x + 2; x \neq -3$

9. $\frac{2b + 4}{4}$ $\frac{b + 2}{2}$

10. $\frac{3a + 15}{15}$ $\frac{a + 5}{5}$

11. $\frac{3p - 21}{18}$ $\frac{p - 7}{6}$

12. $\frac{4}{4y - 8}$ $\frac{1}{y - 2}; y \neq 2$

13. $\frac{7z - 28}{14z}$ $\frac{z - 4}{2z}; z \neq 0$

14. $\frac{9}{18 - 81a}$ $\frac{1}{2 - 9a}; a \neq \frac{2}{9}$

15. $\frac{5}{35 - 5c}$ $\frac{1}{7 - c}; c \neq 7$

16. $\frac{2q + 2}{q^2 + 4q + 3}$ $\frac{2}{q + 3}; q \neq -3$ or -1

17. $\frac{a + 2}{a^2 + 4a + 4}$ $\frac{1}{a + 2}; a \neq -2$

18. $\frac{2x - 2}{2 - 2x}$ $-1; x \neq 1$

19. $\frac{9 - x^2}{x - 3}$ $-x - 3, x \neq 3$

20. $\frac{2a + 4}{2}$ $a + 2$

Write the opposite expression and simplify the opposite expression.

21. $\frac{10b^5}{40b^4}$ $-\frac{10b^5}{40b^4}; -\frac{b}{4}, b \neq 0$

22. $\frac{36 - z^2}{4z - 24}$ $\frac{z^2 - 36}{4z - 24}; \frac{z + 6}{4}, z \neq 6$

23. $\frac{x^2 - 16}{x - 4}$ $\frac{-x^2 + 16}{x - 4}; -x - 4, x \neq 4$

24. $\frac{30 + 2z}{14 + 4z}$ $\frac{-(30 + 2z)}{14 + 4z}; \frac{-15 - z}{7 + 2z}, z \neq 3.5$

Name ______________________________ Class ____________ Date ____________

Reteaching

Multiplying and Dividing Rational Expressions

There are many types of *complex fractions*.

A complex fraction can be a fraction with one or more additional fractions in the numerator, or in the denominator, or in both the numerator and the denominator.

Problem

Is $\dfrac{5x^3}{\frac{6x^2}{x+1}}$ a complex fraction? Explain.

Solve

Ask: Is the numerator a fraction? → No. $5x^3$ is not a fraction.

Ask: Is the denominator a fraction? → Yes. $\frac{6x^2}{x+1}$ is a fraction.

A fraction is in the denominator → $\dfrac{5x^3}{\frac{6x^2}{x+1}}$ is a complex fraction.

Exercises

Tell if the following terms are complex fractions. Explain your reasoning.

1. $\dfrac{\frac{4y}{5}}{\frac{9}{2y}}$ **yes; It has a fraction in the numerator and the denominator.**

2. $\dfrac{2}{3 + 8z}$ **no; It does not have a fraction in the numerator or the denominator.**

3. $\dfrac{\frac{1}{x+2}}{x-2}$ **yes; It has a fraction in the numerator.**

4. $\dfrac{\frac{2x}{3}}{5x}$ **yes; It has a fraction in the numerator.**

5. $\dfrac{\frac{3x^2}{x+8}}{x^3}$ **yes; It has a fraction in the numerator.**

6. $\dfrac{\frac{4x+9}{2x+8}}{\frac{5x-6}{3x+7}}$ **yes; It has a fraction in the numerator and the denominator.**

7. $\dfrac{\frac{x-2}{7}}{x+4}$ **yes; It has a fraction in the numerator.**

8. $\dfrac{\frac{2}{x}}{\frac{x}{5}}$ **yes; It has a fraction in the numerator and the denominator.**

9. $\dfrac{\frac{13x^2}{x+2}}{\frac{4x^3}{x+16}}$ **yes; It has a fraction in the numerator and the denominator.**

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

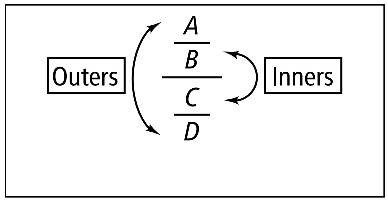
Multiplying and Dividing Rational Expressions

Simplifying Complex Fractions

You can use the *Outers Over Inners* method to simplify complex fractions.

The Outers Over Inners method sets up a simplified fraction that looks like this:

$$\frac{\text{Product of Outers}}{\text{Product of Inners}} \rightarrow \frac{\text{Outers}}{\text{Inners}} = \frac{AD}{BC}$$



For example, in the fraction: $\dfrac{\frac{6y}{5}}{\frac{2}{4y}}$ $6y$ and $4y$ are the "outer" terms; 5 and 2 are the "inner" terms.

If a numerator or denominator is not a fraction, make it a fraction by rewriting it as $\dfrac{\text{polynomial}}{1}$.

Problem

Simplify $\dfrac{\frac{6y}{5}}{\frac{2}{4y}}$.

Solve $\dfrac{\text{Outers}}{\text{Inners}} = \dfrac{(6y)(4y)}{(5)(2)} = \dfrac{24y^2}{10}$

Check Rewrite as numerator divided by denominator. $\dfrac{6y}{5} \div \dfrac{2}{4y}$

Rewrite as a multiplication problem. $\dfrac{6y}{5} \times \dfrac{4y}{2} = \dfrac{24y^2}{10}$

Exercises

Simplify using the Outers Over Inners method.

10. $\dfrac{\frac{(g+4)}{2}}{g}$ $\dfrac{(g+4)}{2g}$

11. $\dfrac{\frac{(x+1)}{2}}{\frac{x}{3}}$ $\dfrac{3(x+1)}{2x}$

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Adding and Subtracting Rational Expressions

Exercises

Add.

1. $\frac{4}{p} + \frac{9}{p}$ $\frac{13}{p}$

2. $\frac{2.5}{x} + \frac{1.25}{x}$ $\frac{3.75}{x}$

3. $\frac{2y}{x^2y^2} + \frac{x^2}{x^2y^2}$ $\frac{x^2 + 2y}{x^2y^2}$

4. $\frac{j^2}{jm^2n} + \frac{3m}{jm^2n} + \frac{2n^3}{jm^2n}$ $\frac{j^2 + 3m + 2n^3}{jm^2n}$

5. $\frac{1}{2a} + \frac{2}{a}$ $\frac{5}{2a}$

6. $\frac{7}{3d} + \frac{3}{7d}$ $\frac{58}{21d}$

7. $\frac{1}{12m} + \frac{3}{4m}$ $\frac{5}{6m}$

8. $\frac{3}{7s} + \frac{11}{4s}$ $\frac{89}{28s}$

9. $\frac{8.9}{16t} + \frac{2.1}{9t}$ $\frac{113.7}{144t}$

10. $\frac{5}{x} + \frac{x}{x^2}$ $\frac{6}{x}$

11. $\frac{3}{n+1} + \frac{4}{n+2}$ $\frac{7n + 10}{(n + 1)(n + 2)}$

12. $\frac{2x}{x^2 - 9} + \frac{4x + 1}{x + 3}$ $\frac{4x^2 - 9x - 3}{x^2 - 9}$

Subtract.

13. $\frac{2}{y} - \frac{5}{3y}$ $\frac{1}{3y}$

14. $\frac{1}{2x} - \frac{1}{3x}$ $\frac{1}{6x}$

15. $\frac{n-1}{2} - \frac{2n}{4}$ $-\frac{1}{2}$

16. $\frac{3}{7y} - \frac{6}{3y}$ $\frac{-11}{7y}$

17. $\frac{11}{2p} - \frac{1}{12p}$ $\frac{65}{12p}$

18. $\frac{4}{x+2} - \frac{2}{x-8}$ $\frac{2x - 36}{x^2 - 6x - 16}$

19. $\frac{10}{x+5} - \frac{x+1}{2x}$ $\frac{-x^2 + 14x - 5}{2x(x + 5)}$

20. $\frac{3x}{2} - \frac{x}{14x}$ $\frac{(21x - 1)}{14}$

21. $\frac{4}{x+2} - 2(x-2)$ $\frac{-2x^2 + 12}{x + 2}$

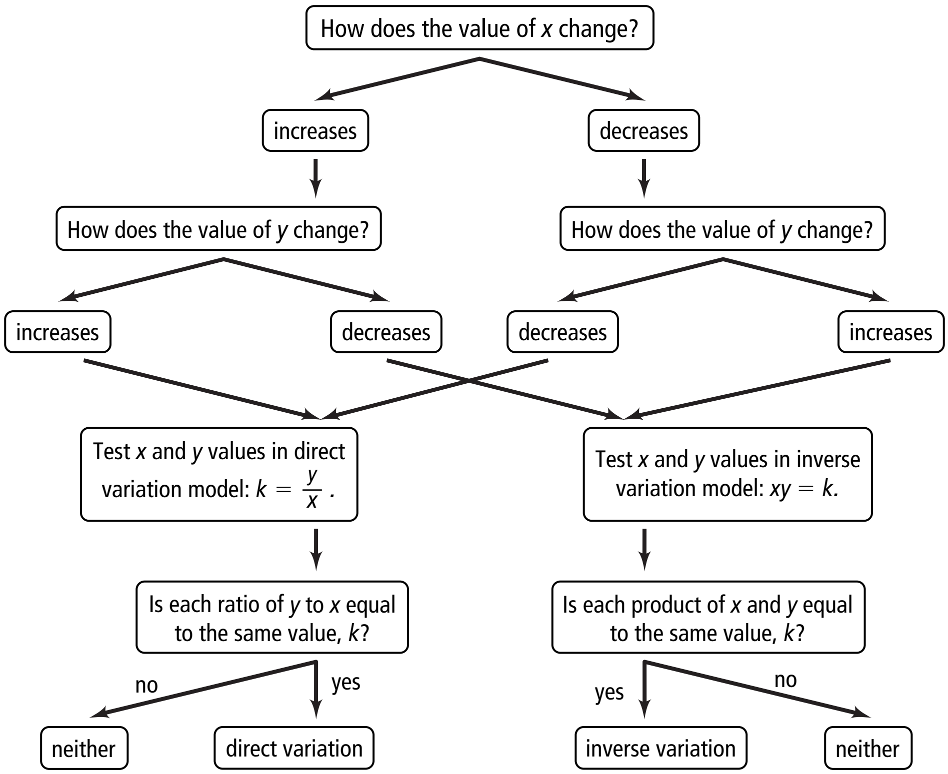
22. $\frac{5k}{2} - \frac{6}{k}$ $\frac{5k^2 - 12}{2k}$

Name ______________________________ Class ____________ Date ____________

Reteaching

Inverse Variation

The flowchart below shows how to decide whether a relationship between two variables is a direct variation, inverse variation, or neither.



Problem

Do the data in the table represent a direct variation, inverse variation, or neither?

x	1	2	4	5
y	20	10	5	4

As the value of x increases, the value of y decreases, so test the table values in the inverse variation model: $xy = k : 1 \cdot 20 = 20$, $2 \cdot 10 = 20$, $4 \cdot 5 = 20$, $5 \cdot 4 = 20$. Each product equals the same value, 20, so the data in the table model an inverse variation.

Exercises

Do the data in the table represent a direct variation, inverse variation, or neither?

1.

x	5	10	15	20
y	10	20	30	40

direct variation

2.

x	1	3	4	6
y	12	4	3	2

inverse variation

Name ______________________________ Class ____________ Date ____________

Reteaching (continued)

Inverse Variation

To solve problems involving inverse variation, you need to solve for the constant of variation k before you can find an answer.

Problem

The time t that is necessary to complete a task varies inversely as the number of people p working. If it takes 4 h for 12 people to paint the exterior of a house, how long does it take for 3 people to do the same job?

$t = \frac{k}{p}$	Write an inverse variation. Because time is dependent on people, t is the dependent variable and p is the independent variable.
$4 = \frac{k}{12}$	Substitute 4 for t and 12 for p.
$48 = k$	Multiply both sides by 12 to solve for k, the constant of variation.
$t = \frac{48}{p}$	Substitute 48 for k. This is the equation of the inverse variation.
$t = \frac{48}{3} = 16$	Substitute 3 for p. Simplify to solve the equation.

It takes 3 people 16 h to paint the exterior of the house.

Exercises

3. The time t needed to complete a task varies inversely as the number of people p. It takes 5 h for seven men to install a new roof. How long does it take ten men to complete the job? **3.5 h**

4. The time t needed to drive a certain distance varies inversely as the speed r. It takes 7.5 h at 40 mi/h to drive a certain distance. How long does it take to drive the same distance at 60 mi/h? **5 h**

5. The cost of each item bought is inversely proportional to the number of items when spending a fixed amount. When 42 items are bought, each costs $1.46. Find the number of items when each costs $2.16. **about 28 items**

6. The length l of a rectangle of a certain area varies inversely as the width w. The length of a rectangle is 9 cm when the width is 6 cm. Determine the length if the width is 8 cm. **6.75 cm**

Name ______________________ Class ____________ Date ____________

Reteaching

The Reciprocal Function Family

A Reciprocal Function in General Form

The *general form* is $y = \frac{a}{x - h} + k$, where $a \neq 0$ and $x \neq h$.

The graph of this equation has a horizontal asymptote at $y = k$ and a vertical asymptote at $x = h$.

Two Members of the Reciprocal Function Family

When $a \neq 1$, $h = 0$, and $k = 0$, you get the *inverse variation function*, $y = \frac{a}{x}$.

When $a = 1$, $h = 0$, and $k = 0$, you get the *parent reciprocal function*, $y = \frac{1}{x}$.

Problem

What is the graph of the inverse variation function $y = \frac{-5}{x}$?

Step 1 Rewrite in general form and identify a, h, and k.

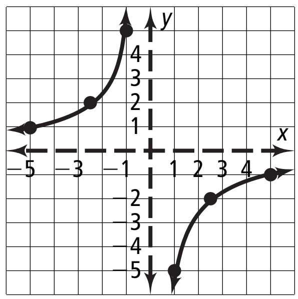
$y = \frac{-5}{x - 0} + 0 \qquad a = -5, h = 0, k = 0$

Step 2 Identify and graph the horizontal and vertical asymptotes.

horizontal asymptote: $y = k$
$y = 0$
vertical asymptote: $x = h$
$x = 0$

Step 3 Make a table of values for $y = \frac{-5}{x}$. Plot the points and then connect the points in each quadrant to make a curve.

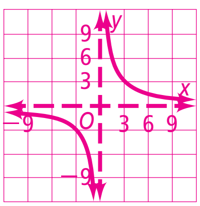
x	−5	−2.5	−1	1	2.5	5
y	1	2	5	−5	−2	−1



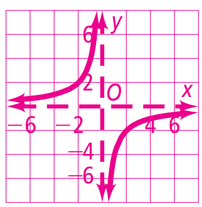
Exercises

Graph each function. Include the asymptotes.

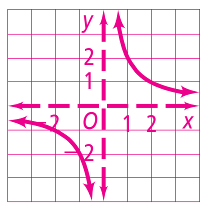
1. $y = \frac{9}{x}$



2. $y = -\frac{4}{x}$



3. $xy = 2$



Name ______________________ Class ____________ Date ____________

Reteaching (continued)

The Reciprocal Function Family

A reciprocal function in the form $y = \frac{a}{x-h} + k$ is a *translation* of the inverse variation function $y = \frac{a}{x}$. The translation is h units horizontally and k units vertically. The translated graph has asymptotes at $x = h$ and $y = k$.

Problem

What is the graph of the reciprocal function $y = -\frac{6}{x+3} + 2$?

Step 1 Rewrite in general form and identify a, h, and k.

$y = \frac{-6}{x-(-3)} + 2 \qquad a = -6,\ h = -3,\ k = 2$

Step 2 Identify and graph the horizontal and vertical asymptotes.

horizontal asymptote: $y = k$

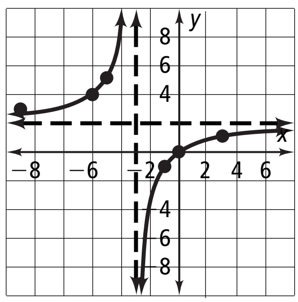
$y = 2$

vertical asymptote: $x = h$

$x = -3$

Step 3 Make a table of values for $y = \frac{-6}{x}$, then *translate* each (x, y) pair to $(x + h, y + k)$. Plot the translated points and connect the points in each quadrant to make a curve.

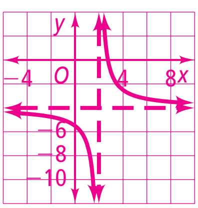
x	−6	−3	−2	2	3	6
y	1	2	3	−3	−2	−1
$x + (-3)$	−9	−6	−5	−1	0	3
$y + 2$	3	4	5	−1	0	1



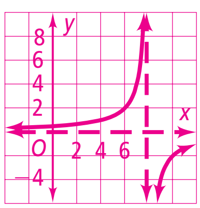
Exercises

Graph each function. Include the asymptotes.

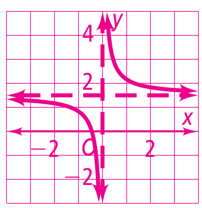
4. $y = \frac{3}{x-2} - 4$



5. $y = -\frac{4}{x-8}$



6. $y = \frac{2}{3x} + \frac{3}{2}$



Name ______________________________ Class ____________ Date ____________

Reteaching

Rational Functions and Their Graphs

A rational function may have one or more types of discontinuities: holes (removable points of discontinuity), vertical asymptotes (non-removable points of discontinuity), or a horizontal asymptote.

If	Then	Example
a is a zero with multiplicity m in the numerator and multiplicity n in the denominator, and $m \geq n$	hole at $x = a$	$f(x) = \frac{(x-5)(x+6)}{(x-5)}$ hole at $x = 5$
a is a zero of the denominator only, or a is a zero with multiplicity m in the numerator and multiplicity n in the denominator, and $m < n$	vertical asymptote at $x = a$	$f(x) = \frac{x^2}{x-3}$ vertical asymptote at $x = 3$

Let p = degree of numerator.
Let q = degree of denominator.

• $p < q$	horizontal asymptote at $y = 0$	$f(x) = \frac{4x^2}{7x^2+2}$ horizontal asymptote at $y = \frac{4}{7}$
• $p > q$	no horizontal asymptote exists	
• $p = q$	horizontal asymptote at $y = \frac{a}{b}$, where a and b are coefficients of highest degree terms in numerator and denominator	

Problem

What are the points of discontinuity of $y = \frac{x^2 + x - 6}{3x^2 - 12}$, if any?

Step 1 Factor the numerator and denominator completely. $y = \frac{(x-2)(x+3)}{3(x-2)(x+2)}$

Step 2 Look for values that are zeros of both the numerator and the denominator. The function has a hole at $x = 2$.

Step 3 Look for values that are zeros of the denominator only. The function has a vertical asymptote at $x = -2$.

Step 4 Compare the degrees of the numerator and denominator. They have the same degree. The function has a horizontal asymptote at $y = \frac{1}{3}$.

Exercises

Find the vertical asymptotes, holes, and horizontal asymptote for the graph of each rational function.

1. $y = \frac{x}{x^2 - 9}$

vertical asymptotes: $x = 3$, $x = -3$; horizontal asymptote: $y = 0$

2. $y = \frac{6x^2 - 6}{x - 1}$

hole: $x = 1$

3. $y = \frac{4x + 5}{3x + 2}$

vertical asymptote: $x = -\frac{2}{3}$; horizontal asymptote: $y = \frac{4}{3}$

Name ______________________________ Class ____________ Date ____________

Reteaching (continued)

Rational Functions and Their Graphs

Before you try to sketch the graph of a rational function, get an idea of its general shape by identifying the graph's holes, asymptotes, and intercepts.

Problem

What is the graph of the rational function $y = \frac{x+3}{x+1}$?

Step 1 Identify any holes or asymptotes.

no holes; vertical asymptote at $x = -1$; horizontal asymptote at $y = \frac{1}{1} = 1$

Step 2 Identify any x- and y-intercepts.

x-intercepts occur when $y = 0$. y-intercepts occur when $x = 0$.

$\frac{x+3}{x+1} = 0$

$x + 3 = 0$

$x = -3$

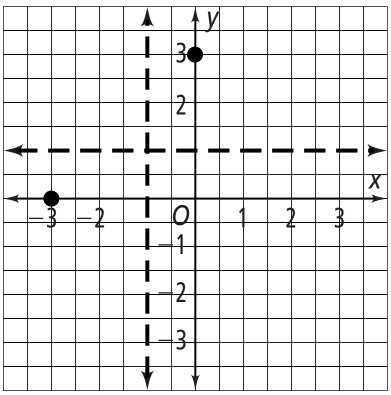
x-intercept at -3

$y = \frac{0+3}{0+1}$

$y = 3$

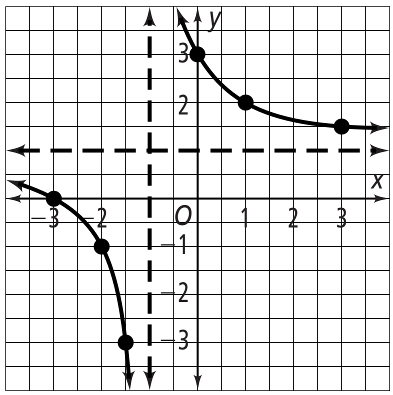
y-intercept at 3

Step 3 Sketch the asymptotes and intercepts.



Step 4 Make a table of values, plot the points, and sketch the graph.

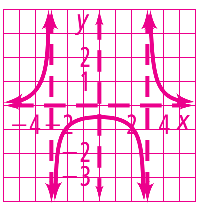
x	y
−2	−1
−1.5	−3
1	2
3	1.5



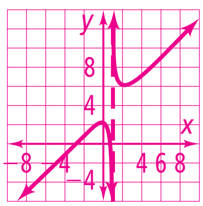
Exercises

Graph each function. Include the asymptotes.

4. $y = \frac{4}{x^2 - 9}$



5. $y = \frac{x^2 + 2x - 2}{x - 1}$



Name ______________________ Class ____________ Date ____________

Reteaching

Solving Rational Equations

When one or both sides of a rational equation has a sum or difference, multiply each side of the equation by the LCD to eliminate the fractions.

Problem

What is the solution of the rational equation $\frac{6}{x} + \frac{x}{2} = 4$? Check the solutions.

$2x\left(\frac{6}{x}\right) + 2x\left(\frac{x}{2}\right) = 2x(4)$	Multiply each term on both sides by the LCD, $2x$.
$2\cancel{x}\left(\frac{6}{\cancel{x}}\right) + \cancel{2}x\left(\frac{x}{\cancel{2}}\right) = 2x(4)$	Divide out the common factors.
$12 + x^2 = 8x$	Simplify.
$x^2 - 8x + 12 = 0$	Write the equation in standard form.
$(x - 2)(x - 6) = 0$	Factor.
$x - 2 = 0$ or $x - 6 = 0$	Use the Zero-Product Property.
$x = 2$ or $x = 6$	Solve for x.

Check

$\frac{6}{x} + \frac{x}{2} \stackrel{?}{=} 4$	$\frac{6}{x} + \frac{x}{2} \stackrel{?}{=} 4$
$\frac{6}{2} + \frac{2}{2} \stackrel{?}{=} 4$	$\frac{6}{6} + \frac{6}{2} \stackrel{?}{=} 4$
$3 + 1 \stackrel{?}{=} 4$	$1 + 3 \stackrel{?}{=} 4$
$4 = 4$ ✓	$4 = 4$ ✓

The solutions are $x = 2$ and $x = 6$.

Exercises

Solve each equation. Check the solutions.

1. $\frac{10}{x+3} + \frac{10}{3} = 6$ $\frac{3}{4}$

2. $\frac{1}{x-3} = \frac{x-4}{x^2-27}$ $\frac{39}{7}$

3. $\frac{6}{x-1} + \frac{2x}{x-2} = 2$ $\frac{8}{5}$

4. $\frac{7}{3x-12} - \frac{1}{x-4} = \frac{2}{3}$ 6

5. $\frac{2x}{5} = \frac{x^2-5x}{5x}$ -5

6. $\frac{8(x-1)}{x^2-4} = \frac{4}{x-2}$ 4

7. $x + \frac{4}{x} = \frac{25}{6}$ $\frac{3}{2}, \frac{8}{3}$

8. $\frac{2}{x} + \frac{6}{x-1} = \frac{6}{x^2-x}$ no solution

9. $\frac{2}{x} + \frac{1}{x} = 3$ 1

10. $\frac{4}{x-1} = \frac{5}{x-1} + 2$ $\frac{1}{2}$

11. $\frac{1}{x} = \frac{5}{2x} + 3$ $-\frac{1}{2}$

12. $\frac{x+6}{5} = \frac{2x-4}{5} - 3$ 25

Reteaching (continued)

Solving Rational Equations

You often can use rational equations to model and solve problems involving rates.

Problem

Quinn can refinish hardwood floors four times as fast as his apprentice, Jack. They are refinishing 100 ft^2 of flooring. Working together, Quinn and Jack can finish the job in 3 h. How long would it take each of them working alone to refinish the floor?

Let x be Jack's work rate in ft^2/h. Quinn's work rate is four times faster, or $4x$.

square feet refinished per hour by Jack and Quinn together	=	square feet of floor they refinish together	÷	hours worked together
ft^2/h	=	ft^2	÷	h

$x + 4x = \frac{100}{3}$	Their work rates sum to 100 ft^2 in 3 h.
$3(x) + 3(4x) = 3\left(\frac{100}{3}\right)$	They work for 3 h. Refinished floor area = rate × time.
$15x = 100$	Simplify.
$x \approx 6.67$	Divide each side by 15.

Jack works at the rate of 6.67 ft^2/h. Quinn works at the rate of 26.67 ft^2/h.

Let j be the number of hours Jack takes to refinish the floor alone, and let q be the number of hours Quinn takes to refinish the floor alone.

$$6.67 = \frac{100}{j} \qquad 26.67 = \frac{100}{q}$$
$$j(6.67) = j\left(\frac{100}{j}\right) \qquad q(26.67) = q\left(\frac{100}{q}\right)$$
$$6.67j = 100 \qquad 26.67q = 100$$
$$j \approx 15 \qquad q \approx 3.75$$

Jack would take 15 h and Quinn would take 3.75 h to refinish the floor alone.

Exercises

13. An airplane flies from its home airport to a city and back in 5 h flying time. The plane travels the 720 mi to the city at 295 mi/h with no wind. How strong is the wind on the return flight? Is the wind a headwind or a tailwind? **about 14 mi/h; headwind**

14. Miguel can complete the decorations for a school dance in 5 days working alone. Nasim can do it alone in 3 days, and Denise can do it alone in 4 days. How long would it take the three students working together to decorate? **about 1.3 days**