Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Solving Systems by Graphing

If the equations represent the same line, there is an infinite number of solutions, the coordinates of any of the points on the line.

Problem

What is the solution to the system? Solve by graphing. Check.

$2x - 3y = 6$

$4x - 6y = 18$

Solution

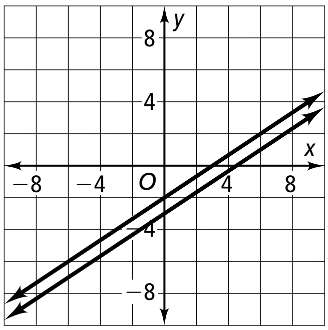
What do you notice about these equations? Using the y-intercepts and solving for the x-intercepts, graph both lines using both sets of points.

$y = \frac{2}{3}x - 2$

$y = \frac{2}{3}x - 3$

Graph equation 1 by finding two points: (0, −2) and (3, 0). Graph equation 2 by finding two points (0, −3) and (4.5, 0).

Is there a solution? Do the lines ever intersect? Lines with the same slope are parallel. Therefore, there is no solution to this system of equations.



Exercises

Solve each system of equations by graphing. Check.

1. $2x = 2 - 9y$
   $21y = 4 - 6x$
   $\left(-\frac{1}{2}, \frac{1}{3}\right)$

2. $2x = 3 - y$
   $y = 4x - 12$
   $\left(\frac{5}{2}, -2\right)$

3. $y = 1.5x + 4$
   $0.5x + y = -2$
   $\left(-3, -\frac{1}{2}\right)$

4. $6y = 2x - 14$
   $x - 7 = 3y$
   infinitely many solutions

5. $3y = -6x - 3$
   $y = 2x - 1$
   (0, −1)

6. $2x = 3y - 12$
   $\frac{1}{3}x = 4y + 5$
   (−9, −2)

7. $2x + 3y = 11$
   $x - y = -7$
   (−2, 5)

8. $3y = 3x - 6$
   $y = x - 2$
   infinitely many solutions

9. $y = \frac{1}{2}x + 9$
   $2y - x = 1$
   no solution

Name ____________________ Class __________ Date __________

Reteaching

Solving Systems Using Substitution

You can solve a system of equations by substituting an equivalent expression for one variable.

Problem

Solve and check the following system:

$$x + 2y = 4$$
$$2x - y = 3$$

Solution

$x + 2y = 4$	The first equation is easiest to solve in terms of one variable.
$x = 4 - 2y$	Get x to one side by subtracting $2y$.
$2(4 - 2y) - y = 3$	Substitute $4 - 2y$ for x in the second equation.
$8 - 4y - y = 3$	Distribute.
$8 - 5y = 3$	Simplify.
$8 - 8 - 5y = 3 - 8$	Subtract 8 from both sides.
$-5y = -5$	Divide both sides by -5.
$y = 1$	You have the solution for y. Solve for x.
$x + 2(1) = 4$	Substitute in 1 for y in the first equation.
$x + 2 - 2 = 4 - 2$	Subtract 2 from both sides.
$x = 2$	The solution is (2, 1).

Check Substitute your solution into either of the given linear equations.

$x + 2y = 4$	
$2 + 2(1) \stackrel{?}{=} 4$	Substitute (2, 1) into the first equation.
$4 = 4$ ✓	You check the second equation.

Exercises

Solve each system using substitution. Check your answer.

1. $x + y = 3$ **(1, 2)**
$2x - y = 0$

2. $x - 3y = -14$ **(4, 6)**
$x - y = -2$

3. $2x - 2y = 10$ **infinitely many solutions**
$x - y = 5$

4. $4x + y = 8$ $\left(\frac{11}{7}, \frac{12}{7}\right)$
$x + 2y = 5$

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Solving Systems Using Substitution

Problem

Solve and check the following system:

$\frac{x}{2} - 3y = 10$

$3x + 4y = -6$

Solve	$\frac{x}{2} - 3y = 10$	First, isolate *x* in the first equation.
	$\frac{x}{2} = 10 + 3y$	Add 3*y* to both sides and simplify.
	$x = 20 + 6y$	Multiply by 2 on both sides.
	$3x + 4y = -6$	Substitute 20 + 6*y* for *x* in second equation.
	$3(20 + 6y) + 4y = -6$	Simplify.
	$60 + 22y = -6$	Subtract 60 from both sides.
	$22y = -66,\ y = -3$	Divide by 22 to solve for *y*.
	$\frac{x}{2} - 3(-3) = 10$	Substitute −3 in the first equation.
	$\frac{x}{2} + 9 = 10$	Simplify.
	$x = 2$	Solve for *x*.

The solution is $(2, -3)$.

Check $3(2) + 4(-3) \stackrel{?}{=} -6$

$-6 = -6$ ✓

Now you check the first equation.

Exercises

Solve each system using substitution. Check your answer.

5. $-2x + y = 8$
$3x + y = -2$ **(−2, 4)**

6. $3x - 4y = 8$
$2x + y = 9$ **(4, 1)**

7. $3x + 2y = 25$
$2x + 3y = -6$ **$(17\frac{2}{5}, -13\frac{3}{5})$**

8. $6x - 5y = 3$
$x - 9y = 25$ **(−2, −3)**

Name ______________________________ Class ____________ Date ____________

Reteaching

Solving Systems Using Elimination

Elimination is one way to solve a system of equations. Think about what the word "eliminate" means. You can eliminate either variable, whichever is easiest.

Problem

Solve and check the following system of linear equations.

$$\begin{aligned} 4x - 3y &= -4 \\ 2x + 3y &= 34 \end{aligned}$$

Solution The equations are already arranged so that like terms are in columns.

Notice how the coefficients of the y-variables have the opposite sign and the same value.

$$\begin{aligned} 4x - 3y &= -4 \\ \underline{2x + 3y} &\underline{= 34} \end{aligned}$$

Add the equations to eliminate y.

$$\begin{aligned} 6x &= 30 \\ x &= 5 \end{aligned}$$

Divide both sides by 6 to solve for x.

$$\begin{aligned} 4(5) - 3y &= -4 \\ 20 - 3y &= -4 \\ -3y &= -24 \\ y &= 8 \end{aligned}$$

Substitute 5 for x in one of the original equations and solve for y.

The solution is (5, 8).

Check

$$\begin{aligned} 4x - 3y &= -4 \\ 4(5) - 3(8) &\stackrel{?}{=} -4 \\ 20 - 24 &\stackrel{?}{=} -4 \\ -4 &= -4 \checkmark \end{aligned}$$

Substitute your solution into both of the original equations to check.

You can check the other equaton.

Exercises

Solve and check each system.

1. $3x + y = 3$ **(0, 3)**
$-3x + y = 3$

2. $6x - 3y = -14$ $\left(\frac{2}{3}, 6\right)$
$6x - y = -2$

3. $3x - 2y = 10$ **(2, −2)**
$x - 2y = 6$

4. $4x + y = 8$ **(1, 4)**
$x + y = 5$

Name ________________ Class ________ Date ________

Reteaching (continued)

Solving Systems Using Elimination

If none of the variables has the same coefficient, you have to multiply before you eliminate.

Problem

Solve the following system of linear equations. $\begin{aligned} -2x - 3y &= -1 \\ 5x + 4y &= 6 \end{aligned}$

Solution

$5(-2x-3y)=(-1)5$ $2(5x+4y)=(6)2$	Multiply the first equation by 5 (all terms, both sides) and the second equation by 2. You can eliminate the *x* variable when you add the equations together.
$-10x-15y=-5$ $\underline{10x+8y=12}$ $-7y=7$	Distribute, simplify and add.
$y=-1$	Divide both sides by 7.
$5x+4(-1)=6$	Substitute –1 in for y in the second equation to find the value of *x*.
$5x-4=6$	Simplify.
$5x=10$	Add 4 to both sides.
$x=2$	Divide by 5 to solve for *x*.

The solution is $(2, -1)$.

Check $-2x-3y=-1$ — Substitute your solution into both original equations.

$-2(2)-3(-1) \stackrel{?}{=} -1$

$-1=-1$ ✓ — You can check the other equation.

Exercises

Solve and check each system.

5. $\begin{aligned} x-3y&=-3 \\ -2x+7y&=10 \end{aligned}$ **(9, 4)**

6. $\begin{aligned} -2x-6y&=0 \\ 3x+11y&=4 \end{aligned}$ **(−6, 2)**

7. $\begin{aligned} 3x+10y&=5 \\ 7x+20y&=11 \end{aligned}$ $\left(1, \frac{1}{5}\right)$

8. $\begin{aligned} 4x+y&=8 \\ x+y&=5 \end{aligned}$ **(1, 4)**

Reteaching (continued)

Applications of Linear Systems

Exercises

1. You have a coin bank that has 275 dimes and quarters that total \$51.50. How many of each type of coin do you have in the bank?
115 dimes; 160 quarters

2. Open-Ended Write a break-even problem and use a system of linear equations to solve it.
Check students' work.

3. You earn a fixed salary working as a sales clerk making \$11 per hour. You get a weekly bonus of \$100. Your expenses are \$60 per week for groceries and \$200 per week for rent and utilities. How many hours do you have to work in order to break even?
about 14.5 h

4. Reasoning Find A and B so that the system below has the solution $(1, -1)$.
$Ax + 2By = 0$
$2Ax - 4By = 16$
$A = 4$; $B = 2$

5. You own an ice cream shop. Your total cost for 12 double cones is \$24 and you sell them for \$2.50 each. How many cones do you have to sell to break even?
10 ice cream cones

6. Multi-Step A skin care cream is made with vitamin C. How many ounces of a 30% vitamin C solution should be mixed with a 10% vitamin C solution to make 50 ounces of a 25% vitamin C solution?

- Define the variables.
- Make a table or drawing to help organize the information.

37.5 oz of 30% solution; 12.5 oz of 10% solution

7. Your hot-air balloon is rising at the rate of 4 feet per second. Another aircraft nearby is at 7452 feet and is losing altitude at the rate of 30 feet per second. In how many seconds will your hot-air balloon be at the same altitude as the other aircraft?
about 219 s

Name ______________________ Class ____________ Date ____________

Reteaching

Linear Inequalities

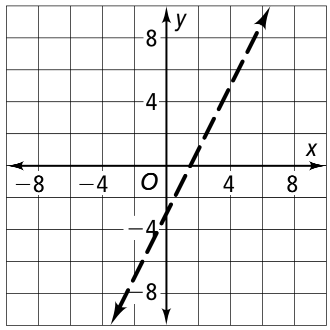
To graph an inequality, graph the line and find the solution region by substituting a test point. The point (0, 0) is a good one unless the line goes through the origin.

Problem

What is the graph of $y > 2x - 3$?

Begin by graphing the line $y = 2x - 3$. Take random values for x, find the corresponding y values, and create a table.

x	$y = 2x - 3$
−2	−7
−1	−5
0	−3
1	−1
2	1



The ordered pairs are $(-2, -7)$, $(-1, -5)$, $(0, -3)$, $(1, -1)$, and $(2, 1)$. You can graph the line using these points. The line should be dashed because the inequality symbol is $>$.

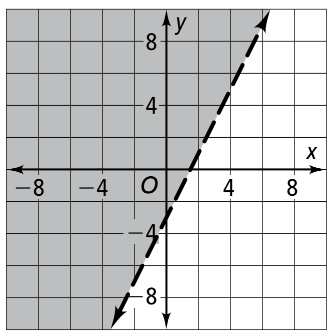
To determine which region to shade, substitute (0, 0) into the inequality to see if it is a solution.

$$y > 2x - 3$$

$$0 \overset{?}{>} 2(0) - 3$$

$$0 > -3 \checkmark$$

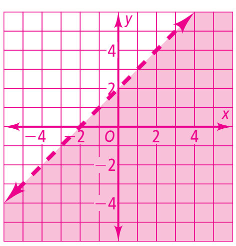
The point (0, 0) satisfies the inequality and is above the line. Therefore, shade the region above the line, which is the solution region.



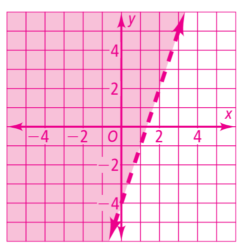
Exercises

Graph each linear inequality.

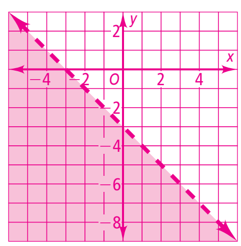
1. $y < x + 2$



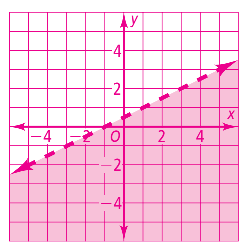
2. $y > 3x - 4$



3. $x + y < -3$



4. $x - 2y > -1$

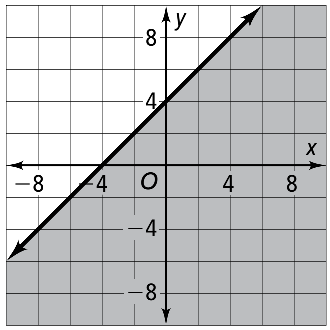


Reteaching (continued)

Linear Inequalities

Problem

What is the inequaltiy for the graph shown?



First look for the y-intercept for the boundary line. The y-intercept is the point (0, 4).

Next determine the slope of the boundary line by finding a second point on the line, $(-4, 0)$. Use the slope formula to determine the slope: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-4)} = \frac{4}{4} = 1$.

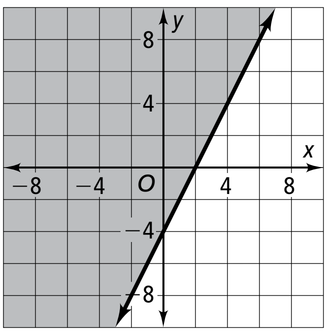
Now you know that the slope is 1 and the y-intercept is 4 and can write an equation for the boundary line $y = x + 4$.

To find the inequality sign, notice that the line is solid. Then note that the shading is below the line, indicating "less than." The inequality is $y \leq x + 4$.

Exercises

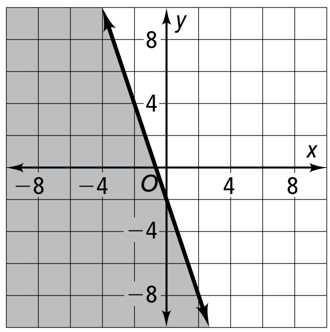
Determine the inequality for each graph shown.

5.



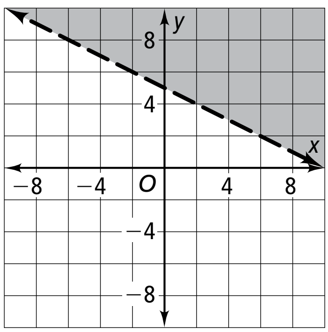
$y \geq 2x - 4$

6.



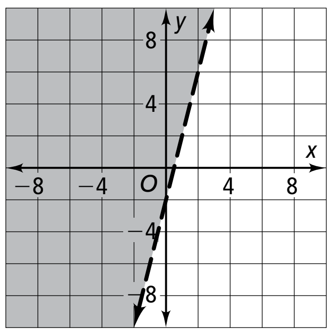
$y \leq -3x - 2$

7.



$y > -\frac{1}{2}x + 5$

8.



$y > 4x - 2$

Name ______________________________ Class ____________ Date ____________

Reteaching (continued)

Systems of Linear Inequalities

Using elimination, solve for q by multiplying all terms in the first equation by -10 and eliminating d: $(q + d < 200)(-10)$.

$$\begin{array}{r} -10q - 10d > -2000 \\ \underline{25q + 10d > 3995} \\ 15q > 1995 \\ q > 133 \end{array}$$

Now add the 2 systems together to solve for q.

$q + d < 200$ Write first inequality.

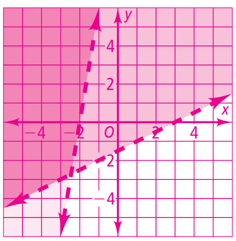
$133 + d < 200,\ d < 67$ Substitute in 133 for q, subtract 133 from both sides and solve for d.

The register contains at least 133 quarters and no more than 67 dimes.

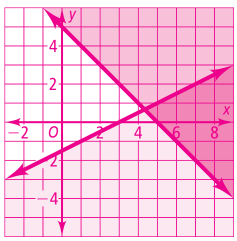
Exercises

Graph the following systems of inequalities.

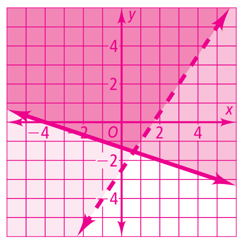
1. $x - 2y < 3$
$\frac{y}{2} > 3x + 6$



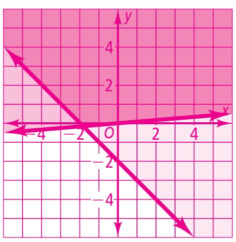
2. $y \geq -x + 5$
$-x \leq -2y - 3$



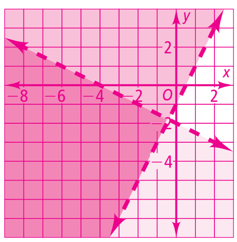
3. $x + 3y \geq -4$
$3x - 2y < 5$



4. $3y \geq \frac{x}{4}$
$-y \leq x + 2$



5. $2x - y < 1$
$x + 2y < -4$



6. $5x - 4y \geq 3$
$2x + 3y \leq -2$