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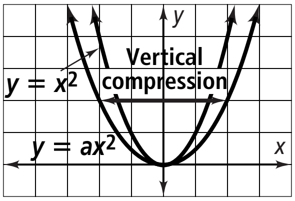
Reteaching (continued)

Quadratic Functions and Transformations

If $a \neq 1$, the graph is a stretch or compression of the parent function by a factor of $|a|$.

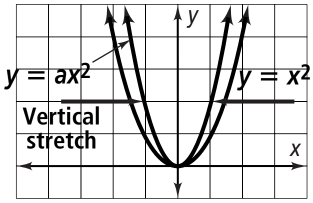
$0 < |a| < 1$

The graph is a vertical compression of the parent function.



$|a| > 1$

The graph is a vertical stretch of the parent function



Problem

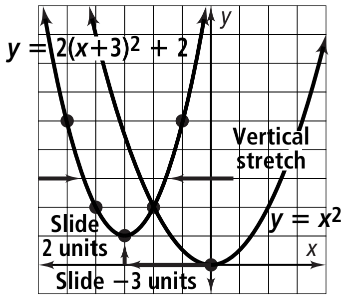
What is the graph of $y = 2(x + 3)^2 + 2$?

Step 1 Write the function in vertex form: $y = 2[x - (-3)]^2 + 2$

Step 2 The vertex is $(-3, 2)$.

Step 3 The axis of symmetry is $x = -3$.

Step 4 Because $a = 2$, the graph of this function is a vertical stretch by 2 of the parent function. In addition to sliding the graph of the parent function 3 units left and 2 units up, you must change the shape of the graph. Plot a few points near the vertex to help you sketch the graph.

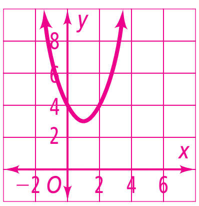


x	−5	−4	−3	−2	−1
y	10	4	2	4	10

Exercises

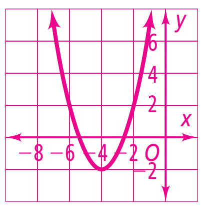
Graph each function. Identify the vertex and axis of symmetry.

1. $y = (x - 1)^2 + 3$



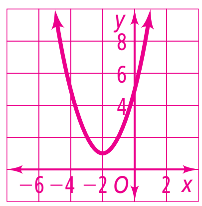
(1, 3); $x = 1$

2. $y = (x + 4)^2 - 2$



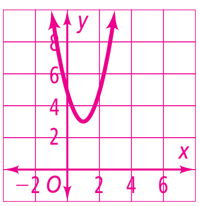
(−4, −2); $x = -4$

3. $y = (x + 2)^2 + 1$



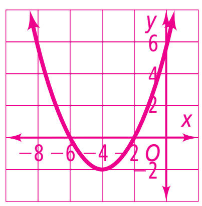
(−2, 1); $x = -2$

4. $y = 2(x - 1)^2 + 3$



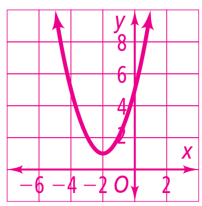
(1, 3); $x = 1$

5. $y = \frac{1}{2}(x + 4)^2 - 2$



(−4, −2); $x = -4$

6. $y = 0.9(x + 2)^2 + 1$



(−2, 1); $x = -2$

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Reteaching

Standard Form of a Quadratic Function

- The graph of a quadratic function, $y = ax^2 + bx + c$, where $a \neq 0$, is a parabola.
- The axis of symmetry is the line $x = -\frac{b}{2a}$.
- The x-coordinate of the vertex is $-\frac{b}{2a}$. The y-coordinate of the vertex is $y = f\left(-\frac{b}{2a}\right)$, or the y-value when $x = -\frac{b}{2a}$.
- The y-intercept is $(0, c)$.

Problem

What is the graph of $y = 2x^2 - 8x + 5$?

$x = -\frac{b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$ — Find the equation of the axis of symmetry.

x-coordinate of vertex: 2 — $-\frac{b}{2a}$

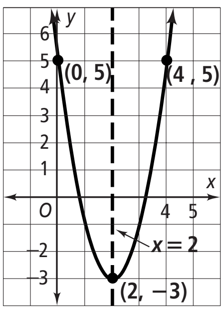
$f\left(-\frac{b}{2a}\right) = f(2) = 2(2)^2 - 8(2) + 5$ — Find the y-value when $x = 2$.

$= 8 - 16 + 5$

$= -3$

y-coordinate of vertex: -3 — The vertex is $(2, -3)$.

y-intercept: $(0, 5)$ — The y-intercept is at $(0, c) = (0, 5)$.

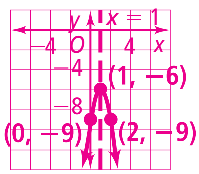


Because a is positive, the graph opens upward, and the vertex is at the bottom of the graph. Plot the vertex and draw the axis of symmetry. Plot (0, 5) and its corresponding point on the other side of the axis of symmetry.

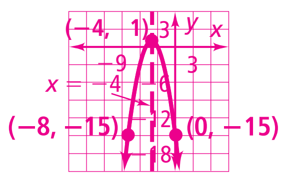
Exercises

Graph each parabola. Label the vertex and the axis of symmetry.

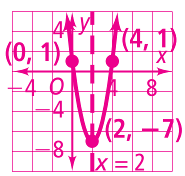
1. $y = -3x^2 + 6x - 9$



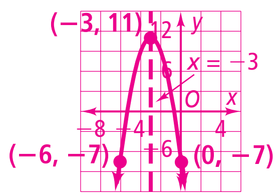
2. $y = -x^2 - 8x - 15$



3. $y = 2x^2 - 8x + 1$



4. $y = -2x^2 - 12x - 7$



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Reteaching (continued)

Standard Form of a Quadratic Function

- Standard form of a quadratic function is $y = ax^2 + bx + c$. Vertex form of a quadratic function is $y = a(x - h)^2 + k$.
- For a parabola in vertex form, the coordinates of the vertex are (h, k).

Problem

What is the vertex form of $y = 3x^2 - 24x + 50$?

$y = ax^2 + bx + c$	
$y = 3x^2 - 24x + 50$	Verify that the equation is in standard form.
$b = -24, \quad a = 3$	Find b and a.
$x\text{-coordinate} = -\frac{b}{2a}$	For an equation in standard form, the x-coordinate of the vertex can be found by using $x = -\frac{b}{2a}$.
$= -\frac{-24}{2(3)}$	Substitute.
$= 4$	Simplify.
$y\text{-coordinate} = 3(4)^2 - 24(4) + 50$	Substitute 4 into the standard form to find the y-coordinate.
$= 2$	Simplify.
$y = 3(x - 4)^2 + 2$	Substitute 4 for h and 2 for k into the vertex form.

Once the conversion to vertex form is complete, check by multiplying.

$y = 3(x^2 - 8x + 16) + 2$

$y = 3x^2 - 24x + 50$

The result is the standard form of the equation.

Exercises

Write each function in vertex form. Check your answers.

5. $y = x^2 - 2x - 3$
$y = (x - 1)^2 - 4$

6. $y = -x^2 + 4x + 6$
$y = -(x - 2)^2 + 10$

7. $y = x^2 + 3x - 10$
$y = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$

8. $y = x^2 - 9x$
$y = \left(x - \frac{9}{2}\right)^2 - \frac{81}{4}$

9. $y = x^2 + x$
$y = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$

10. $y = x^2 + 5x + 4$
$y = \left(x + \frac{5}{2}\right)^2 - \frac{9}{4}$

11. $y = 4x^2 + 8x - 3$
$y = 4(x + 1)^2 - 7$

12. $y = \frac{3}{4}x^2 + 9x$
$y = \frac{3}{4}(x + 6)^2 - 27$

13. $y = -2x^2 + 2x + 1$
$y = -2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$

Write each function in standard form.

14. $y = (x - 3)^2 + 1$
$y = x^2 - 6x + 10$

15. $y = 2(x - 1)^2 - 3$
$y = 2x^2 - 4x - 1$

16. $y = -3(x + 4)^2 + 1$
$y = -3x^2 - 24x - 47$

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Reteaching

Factoring Quadratic Expressions

Problem

What is $6x^2 - 5x - 4$ in factored form?

$a = 6$, $b = -5$, and $c = -4$ — Find *a*, *b*, and *c*; they are the coefficients of each term.

$ac = -24$ and $b = -5$ — We are looking for factors with product *ac* and sum *b*.

Factors of −24	1, −24	−1, 24	2, −12	−2, 12	3, −8	−3, 8	4, −6	−4, 6
Sum of factors	−23	23	−10	10	−5	5	−2	2

The factors 3 and −8 are the combination whose sum is −5.

$\underbrace{6x^2 + 3x} - \underbrace{8x - 4}$ — Rewrite the middle term using the factors you found.

$3x(2x + 1) - 4(2x + 1)$ — Find common factors by grouping the terms in pairs.

$(3x - 4)(2x + 1)$ — Rewrite using the Distributive Property.

Check $(3x - 4)(2x + 1)$ — You can check your answer by multiplying the factors together.

$6x^2 + 3x - 8x - 4$

$6x^2 - 5x - 4$

Remember that not all quadratic expressions are factorable.

Exercises

Factor each expression.

1. $x^2 + 6x + 8$ **$(x + 4)(x + 2)$**
2. $x^2 - 4x + 3$ **$(x - 3)(x - 1)$**
3. $2x^2 - 6x + 4$ **$2(x - 2)(x - 1)$**
4. $2x^2 - 11x + 5$ **$(2x - 1)(x - 5)$**
5. $2x^2 - 7x - 4$ **$(2x + 1)(x - 4)$**
6. $4x^2 + 16x + 15$ **$(2x + 5)(2x + 3)$**
7. $x^2 - 5x - 14$ **$(x + 2)(x - 7)$**
8. $7x^2 - 19x - 6$ **$(7x + 2)(x - 3)$**
9. $x^2 - x - 72$ **$(x - 9)(x + 8)$**
10. $2x^2 + 9x + 7$ **$(2x + 7)(x + 1)$**
11. $x^2 + 12x + 32$ **$(x + 4)(x + 8)$**
12. $4x^2 - 28x + 49$ **$(2x - 7)(2x - 7)$**
13. $x^2 - 3x - 10$ **$(x - 5)(x + 2)$**
14. $2x^2 + 9x + 4$ **$(2x + 1)(x + 4)$**
15. $9x^2 - 6x + 1$ **$(3x - 1)(3x - 1)$**
16. $x^2 - 10x + 9$ **$(x - 1)(x - 9)$**
17. $x^2 + 4x - 12$ **$(x + 6)(x - 2)$**
18. $x^2 + 7x + 10$ **$(x + 5)(x + 2)$**
19. $x^2 - 8x + 12$ **$(x - 6)(x - 2)$**
20. $2x^2 - 5x - 3$ **$(2x + 1)(x - 3)$**
21. $x^2 - 6x + 5$ **$(x - 1)(x - 5)$**
22. $3x^2 + 2x - 8$ **$(3x - 4)(x + 2)$**

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Reteaching (continued)

Factoring Quadratic Expressions

- $a^2 + 2ab + b^2 = (a + b)^2$ Factoring perfect square trinomials
  $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a + b)(a - b)$ Factoring a difference of two squares

Problem

What is $25x^2 - 20x + 4$ in factored form?

There are three terms. Therefore, the expression may be a perfect square trinomial.

$a^2 = 25x^2$ and $b^2 = 4$ Find a^2 and b^2.

$a = 5x$ and $b = 2$ Take square roots to find a and b.

Check that the choice of a and b gives the correct middle term.

$2ab = 2 \cdot 5x \cdot 2 = 20x$

Write the factored form.

$a^2 - 2ab + b^2 = (a - b)^2$

$25x^2 - 20x + 4 = (5x - 2)^2$

Check $(5x - 2)^2$ You can check your answer by multiplying the factors together.

$(5x - 2)(5x - 2)$ Rewrite the square in expanded form.

$25x^2 - 10x - 10x + 4$ Distribute.

$25x^2 - 20x + 4$ Simplify.

Exercises

Factor each expression.

23. $x^2 - 12x + 36$
$(x - 6)^2$

24. $x^2 + 30x + 225$
$(x + 15)^2$

25. $9x^2 - 12x + 4$
$(3x - 2)^2$

26. $x^2 - 64$
$(x + 8)(x - 8)$

27. $9x^2 - 42x + 49$
$(3x - 7)^2$

28. $25x^2 - 1$
$(5x + 1)(5x - 1)$

29. $27x^2 - 12$
$3(3x + 2)(3x - 2)$

30. $49x^2 + 42x + 9$
$(7x + 3)^2$

31. $16x^2 - 32x + 16$
$16(x - 1)^2$

32. $9x^2 - 16$
$(3x + 4)(3x - 4)$

33. $8x^2 - 18$
$2(2x + 3)(2x - 3)$

34. $81x^2 + 126x + 49$
$(9x + 7)^2$

35. $125x^2 - 100x + 20$
$5(5x - 2)^2$

36. $-x^2 + 196$
$-(x + 14)(x - 14)$

37. $-16x^2 - 24x - 9$
$-(4x + 3)^2$

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Reteaching

Quadratic Equations

There are several ways to solve quadratic equations. If you can factor the quadratic expression in a quadratic equation written in standard form, you can use the Zero-Product Property.

If $ab = 0$ then $a = 0$ or $b = 0$.

Problem

What are the solutions of the quadratic equation $2x^2 + x = 15$?

$2x^2 + x = 15$	Write the equation.
$2x^2 + x - 15 = 0$	Rewrite in standard form, $ax^2 + bx + c = 0$.
$(2x - 5)(x + 3) = 0$	Factor the quadratic expression (the nonzero side).
$2x - 5 = 0$ or $x + 3 = 0$	Use the Zero-Product Property.
$2x = 5$ or $x = -3$	Solve for x.
$x = \frac{5}{2}$ or $x = -3$	

Check the solutions:

$x = \frac{5}{2}$: $2\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right) \stackrel{?}{=} 15$

$\frac{25}{2} + \frac{5}{2} \stackrel{?}{=} 15$

$15 = 15$

$x = -3$: $2(-3)^2 + (-3) \stackrel{?}{=} 15$

$18 - 3 \stackrel{?}{=} 15$

$15 = 15$

Both solutions check. The solutions are $x = \frac{5}{2}$ and $x = -3$.

Exercises

Solve each equation by factoring. Check your answers.

1. $x^2 - 10x + 16 = 0$ **2, 8**
2. $x^2 + 2x = 63$ **-9, 7**
3. $x^2 + 9x = 22$ **-11, 2**
4. $x^2 - 24x + 144 = 0$ **12**
5. $2x^2 = 7x + 4$ **$-\frac{1}{2}$, 4**
6. $2x^2 = -5x + 12$ **-4, $\frac{3}{2}$**
7. $x^2 - 7x = -12$ **3, 4**
8. $2x^2 + 10x = 0$ **-5, 0**
9. $x^2 + x = 2$ **-2, 1**
10. $3x^2 - 5x + 2 = 0$ **$\frac{2}{3}$, 1**
11. $x^2 = -5x - 6$ **-3, -2**
12. $x^2 + x = 20$ **-5, 4**

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Reteaching (continued)

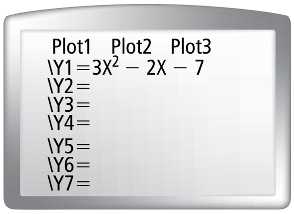
Quadratic Equations

Some quadratic equations are difficult or impossible to solve by factoring. You can use a graphing calculator to find the points where the graph of a function intersects the x-axis. At these points $f(x) = 0$, so x is a zero of the function.

The values r_1 and r_2 are the zeros of the function $y = (x - r_1)(x - r_2)$. The graph of the function intersects the x-axis at $x = r_1$, or $(r_1, 0)$, and $x = r_2$, or $(r_2, 0)$.

Problem

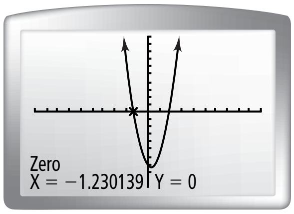
What are the solutions of the quadratic equation $3x^2 = 2x + 7$?



Step 1 Rewrite the equation in standard form, $ax^2 + bx + c = 0$.
$3x^2 - 2x - 7 = 0$

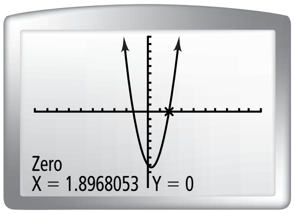
Step 2 Enter the equation as Y1 in your calculator.

Step 3 Graph Y1. Choose the standard window and see if the zeros of the function Y1 are visible on the screen. If they are not visible, zoom out and determine a better viewing window. In this case, the zeros are visible in the standard window.



Step 4 Use the ZERO option in the CALC feature. For the first zero, choose bounds of -2 and -1 and a guess of -1.5. The screen display gives the first zero as $x = -1.230139$.

Similarly, the screen display gives the second zero as $x = 1.8968053$.



The solutions to two decimal places are $x = -1.23$ and $x = 1.90$.

Exercises

Solve the equation by graphing. Give each answer to at most two decimal places.

13. $x^2 = 5$ **−2.24, 2.24**

14. $x^2 = 5x + 1$ **−0.19, 5.19**

15. $x^2 + 7x = 3$ **−7.41, 0.41**

16. $x^2 + x = 5$ **−2.79, 1.79**

17. $x^2 + 3x + 1 = 0$ **−2.62, −0.38**

18. $x^2 = 2x + 4$ **−1.24, 3.24**

19. $3x^2 - 5x + 9 = 8$ **0.23, 1.43**

20. $4 = 2x^2 + 3x$ **−2.35, 0.85**

21. $x^2 - 6x = -7$ **1.59, 4.41**

22. $-x^2 = 8x + 8$ **−6.83, −1.17**

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Reteaching

Completing the Square

Completing a perfect square trinomial allows you to factor the completed trinomial as the square of a binomial.

Start with the expression $x^2 + bx$. Add $\left(\frac{b}{2}\right)^2$. Now the expression is $x^2 + bx + \left(\frac{b}{2}\right)^2$, which can be factored into the square of a binomial: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$.

To complete the square for an expression $ax^2 + abx$, first factor out a. Then find the value that completes the square for the factored expression.

Problem

What value completes the square for $-2x^2 + 10x$?

Think	Write
Write the expression in the form $a(x^2 + bx)$.	$-2x^2 + 10x = -2(x^2 - 5x)$
Find $\frac{b}{2}$.	$\frac{b}{2} = \frac{-5}{2} = -\frac{5}{2}$
Add $\left(\frac{b}{2}\right)^2$ to the inner expression to complete the square.	$-2\left[x^2 - 5x + \left(-\frac{5}{2}\right)^2\right] = -2\left(x^2 - 5x + \frac{25}{4}\right)$
Factor the perfect square trinomial.	$-2\left(x - \frac{5}{2}\right)^2$
Find the value that completes the square.	$-2\left(\frac{25}{4}\right) = -\frac{25}{2}$

Exercises

What value completes the square for each expression?

1. $x^2 + 2x$ **1**

2. $x^2 - 24x$ **144**

3. $x^2 + 12x$ **36**

4. $x^2 - 20x$ **100**

5. $x^2 + 5x$ $\frac{25}{4}$

6. $x^2 - 9x$ $\frac{81}{4}$

7. $2x^2 - 24x$ **72**

8. $3x^2 + 12x$ **12**

9. $-x^2 + 6x$ **−9**

10. $5x^2 + 80x$ **320**

11. $-7x^2 + 14x$ **−7**

12. $-3x^2 - 15x$ $-\frac{75}{4}$

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Reteaching (continued)

Completing the Square

You can easily graph a quadratic function if you first write it in vertex form. Complete the square to change a function in standard form into a function in vertex form.

Problem

What is $y = x^2 - 6x + 14$ in vertex form?

Think	Write
Write an expression using the terms that contain x.	$x^2 - 6x$
Find $\frac{b}{2}$.	$\frac{b}{2} = \frac{-6}{2} = -3$
Add $\left(\frac{b}{2}\right)^2$ to the expression to complete the square.	$x^2 - 6x + (-3)^2 = x^2 - 6x + 9$
Subtract 9 from the expression so that the equation is unchanged.	$y = x^2 - 6x + 9 + 14 - 9$
Factor the perfect square trinomial.	$y = (x - 3)^2 + 14 - 9$
Add the remaining constant terms.	$y = (x - 3)^2 + 5$

Exercises

Rewrite each equation in vertex form.

13. $y = x^2 + 4x + 3$ **$(x + 2)^2 - 1$**

14. $y = x^2 - 6x + 13$ **$(x - 3)^2 + 4$**

15. $y = -x^2 + 4x - 10$ **$-(x - 2)^2 - 6$**

16. $y = x^2 - 2x - 3$ **$(x - 1)^2 - 4$**

17. $y = x^2 + 8x + 13$ **$(x + 4)^2 - 3$**

18. $y = -x^2 - 6x - 4$ **$-(x + 3)^2 + 5$**

19. $y = -x^2 + 10x - 18$ **$-(x - 5)^2 + 7$**

20. $y = x^2 + 2x - 8$ **$(x + 1)^2 - 9$**

21. $y = 2x^2 + 4x - 3$ **$2(x + 1)^2 - 5$**

22. $y = 3x^2 - 12x + 8$ **$3(x - 2)^2 - 4$**

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Reteaching

The Quadratic Formula

You can solve some quadratic equations by factoring or completing the square. You can solve any quadratic equation $ax^2 + bx + c = 0$ by using the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

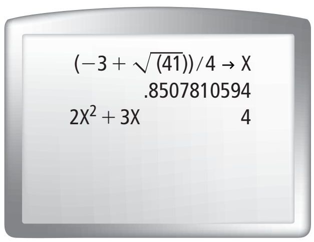
Notice the $\pm$ symbol in the formula. Whenever $b^2 - 4ac$ is not zero, the Quadratic Formula will result in two solutions.

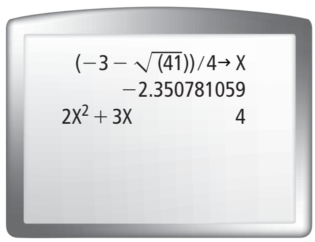
Problem

What are the solutions for $2x^2 + 3x = 4$? Use the Quadratic Formula.

$2x^2 + 3x - 4 = 0$	Write the equation in standard form: $ax^2 + bx + c = 0$
$a = 2;\ b = 3;\ c = -4$	a is the coefficient of x^2, b is the coefficient of x, c is the constant term.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Write the Quadratic Formula.
$= \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-4)}}{2(2)}$	Substitute 2 for a, 3 for b, and –4 for c.
$= \frac{-3 \pm \sqrt{41}}{4}$	Simplify.
$= \frac{-3 + \sqrt{41}}{4}$ or $\frac{-3 - \sqrt{41}}{4}$	Write the solutions separately.

Check your results on your calculator. Replace x in the original equation with $\frac{-3 + \sqrt{41}}{4}$ and $\frac{-3 - \sqrt{41}}{4}$. Both values for x give a result of 4. The solutions check.





Exercises

What are the solutions for each equation? Use the Quadratic Formula.

1. $-x^2 + 7x - 3 = 0$ **$\frac{7 + \sqrt{37}}{2}$ or $\frac{7 - \sqrt{37}}{2}$**

2. $x^2 + 6x = 10$ **$-3 + \sqrt{19}$ or $-3 - \sqrt{19}$**

3. $2x^2 = 4x + 3$ **$\frac{2 + \sqrt{10}}{2}$ or $\frac{2 - \sqrt{10}}{2}$**

4. $4x^2 + 81 = 36x$ **$\frac{9}{2}$**

5. $2x^2 + 1 = 5 - 7x$ **-4 or $\frac{1}{2}$**

6. $6x^2 - 10x + 3 = 0$ **$\frac{5 + \sqrt{7}}{6}$ or $\frac{5 - \sqrt{7}}{6}$**

Reteaching (continued)

The Quadratic Formula

There are three possible outcomes when you take the square root of a real number *n*:

$$n \begin{cases} > 0 & \rightarrow \quad \text{two real values (one positive and one negative)} \\ = 0 & \rightarrow \quad \text{one real value (0)} \\ < 0 & \rightarrow \quad \text{no real values} \end{cases}$$

Now consider the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The value under the radical symbol determines the number of real solutions that exist for the equation $ax^2 + bx + c = 0$:

$$b^2 - 4ac \begin{cases} > 0 & \rightarrow \quad \text{two real solutions} \\ = 0 & \rightarrow \quad \text{one real solution} \\ < 0 & \rightarrow \quad \text{no real solutions} \end{cases}$$

> The value under the radical, $\mathbf{b^2 - 4ac}$, is called the **discriminant**.

Problem

What is the number of real solutions of $-3x^2 + 7x = 2$?

$-3x^2 + 7x = 2$	
$-3x^2 + 7x - 2 = 0$	Write in standard form.
$a = -3, b = 7, c = -2$	Find the values of *a*, *b*, and *c*.
$b^2 - 4ac$	Write the discriminant.
$(7)^2 - 4(-3)(-2)$	Substitute for *a*, *b*, and *c*.
$49 - 24$	Simplify.
25	

The discriminant, 25, is positive. The equation has two real roots.

Exercises

What is the value of the discriminant and what is the number of real solutions for each equation?

7. $x^2 + x - 42 = 0$
169; two

8. $-x^2 + 13x - 40 = 0$
9; two

9. $x^2 + 2x + 5 = 0$
–16; none

10. $x^2 = 18x - 81$
0; one

11. $-x^2 + 7x + 44 = 0$
225; two

12. $\frac{1}{4}x^2 - 5x + 25 = 0$
0; one

13. $2x^2 + 7 = 5x$
–31; none

14. $4x^2 + 25x = 21$
961; two

15. $x^2 + 5 = 3x$
–11; none

16. $\frac{1}{9}x^2 = 4x - 36$
0; one

17. $\frac{1}{2}x^2 + 2x + 3 = 0$
–2; none

18. $\frac{1}{6}x^2 = 2x + 18$
16; two

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Reteaching

Quadratic Systems

You used graphing and substitution to solve systems of linear equations. You can use these same methods to solve systems involving quadratic equations.

Problem

What is the solution of the system of equations? $\begin{cases} y = x^2 - 2x - 8 \\ y = 2x - 3 \end{cases}$

$y = 2x - 3$	Write one equation.
$x^2 - 2x - 8 = 2x - 3$	Substitute $x^2 - 2x - 8$ for y in the linear equation.
$x^2 - 4x - 5 = 0$	Write in standard form.
$(x + 1)(x - 5) = 0$	Factor the quadratic expression.
$x + 1 = 0$ or $x - 5 = 0$	Use the Zero-Product Property.
$x = -1$ or $x = 5$	Solve for x.

Because the solutions to the system of equations are ordered pairs of the form (x, y), solve for y by substituting each value of x into the linear equation. You can use either equation, but the linear equation is easier.

$x = -1$: $y = 2x - 3 = 2(-1) - 3 = -5$ → $(-1, -5)$
$x = 5$: $y = 2x - 3 = 2(5) - 3 = 7$ → $(5, 7)$

The solutions are $(-1, -5)$ and $(5, 7)$. Check these by graphing the system and identifying the points of intersection.

Exercises

Solve each system.

1. $\begin{cases} y = x^2 + 3x - 5 \\ y = 3x - 4 \end{cases}$ **(−1, −7), (1, −1)**

2. $\begin{cases} y = -x^2 + 5x - 1 \\ y = -x + 4 \end{cases}$ **(1, 3), (5, −1)**

3. $\begin{cases} y = 2x^2 - x - 5 \\ y = 3x + 1 \end{cases}$ **(−1, −2), (3, 10)**

4. $\begin{cases} y = x^2 + 3x - 7 \\ y = -x - 2 \end{cases}$ **(−5, 3), (1, −3)**

5. $\begin{cases} y = 2x^2 - 5x + 1 \\ y = 5x - 7 \end{cases}$ **(1, −2), (4, 13)**

6. $\begin{cases} y = -x^2 - 2x + 3 \\ y = x - 1 \end{cases}$ **(−4, −5), (1, 0)**

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Reteaching (continued)

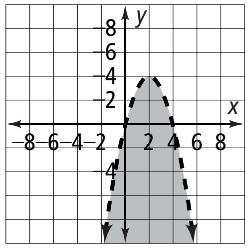
Quadratic Systems

To solve a system of linear inequalities, you graph each inequality and find the region where the graphs overlap. You can also use this technique to solve a system of quadratic inequalities.

Problem

What is the solution of this system of inequalities? $\begin{cases} y < -x^2 + 4x \\ y > x^2 - 2x - 8 \end{cases}$

Step 1 Graph the equation $y = -x^2 + 4x$. Use a dashed boundary line because the points on the curve are not part of the solution. Choose a point on one side of the curve and check if it satisfies the inequality.



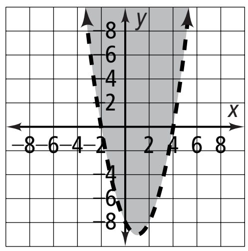
$y < -x^2 + 4x$

$0 \overset{?}{<} -(2)^2 + 4(2)$ Check the point (2, 0).

$0 < 4$ The inequality is true.

Points below the curve satisfy the inequality, so shade that region.

Step 2 Graph the equation $y = x^2 - 2x - 8$. Use a dashed boundary line because the points on the curve are not part of the solution. Choose a point on one side of the curve and check if it satisfies the inequality.



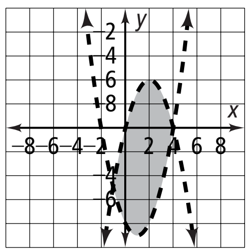
$y > x^2 - 2x - 8$

$0 \overset{?}{>} (2)^2 - 2(2) - 8$ Check the point (2, 0).

$0 > -8$ The inequality is true.

Points above the curve satisfy the inequality, so shade that region.

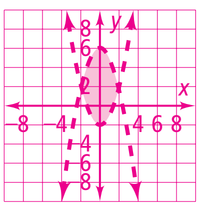
Step 3 The solution to the system of both inequalities is the set of points satisfying both inequalities. In other words, the solution is the region where the graphs overlap. The region contains no boundary points.



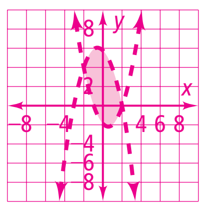
Exercises

Solve each system by graphing.

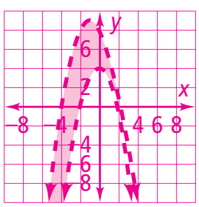
7. $\begin{cases} y < -x^2 + 6 \\ y > x^2 - 2 \end{cases}$



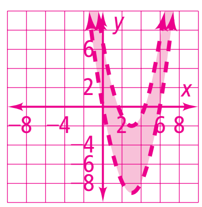
8. $\begin{cases} y > x^2 - x - 2 \\ y < -x^2 - x + 6 \end{cases}$



9. $\begin{cases} y < -x^2 - 2x + 8 \\ y > -x^2 + 4 \end{cases}$



10. $\begin{cases} y > x^2 - 6x \\ y < x^2 - 6x + 7 \end{cases}$



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Reteaching

A New Look at Parabolas

Problem

What is the graph of the equation $y = -\frac{1}{2}x^2$? Label the vertex, focus, and directrix on your graph.

Step 1

Identify information from the given equation.

$y = -\frac{1}{2}x^2$

$a < 0$	a is negative.
opens downward focus: $(0, -c)$ directrix: $y = c$	When a is negative, the parabola has these characteristics.

Step 2

Find c.

$\lvert a\rvert = \frac{1}{4c}$	True for all parabolas.
$\left\lvert -\frac{1}{2}\right\rvert = \frac{1}{4c}$	Substitute $-\frac{1}{2}$ for a.
$(2c)\left\lvert -\frac{1}{2}\right\rvert = (2c)\frac{1}{4c} = \frac{1}{2}$	Solve for c.

Step 3

Find the vertex, the focus, and the equation of the directrix.

$(0, 0)$	The parabola is of the form $y = ax^2$, so the vertex is at the origin.
$\left(0, -\frac{1}{2}\right)$	The focus is always $(0, -c)$.
$y = \frac{1}{2}$	The directrix is at $y = c$.

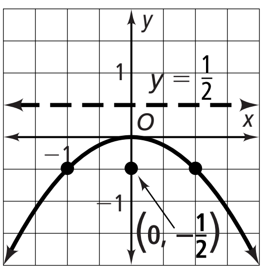
Step 4

Locate two more points on the parabola.

$y = -\frac{1}{2}(1)^2$	Substitute 1 for x.
$y = -\frac{1}{2}$	Solve for y.
$\left(1, -\frac{1}{2}\right)$	
$y = -\frac{1}{2}(-1)^2$	Substitute -1 for x.
$y = -\frac{1}{2}$	Solve for y.
$\left(-1, -\frac{1}{2}\right)$	

Step 5

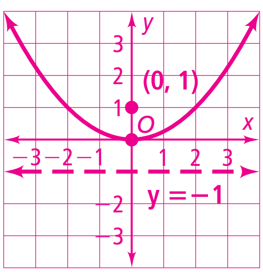
Graph the parabola using the information you found.



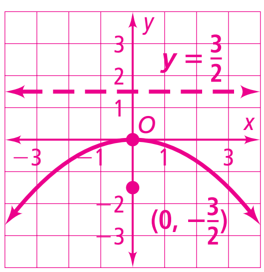
Exercises

Graph each equation. Label the vertex, focus, and directrix on each graph.

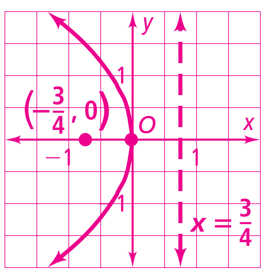
1. $y = \frac{1}{4}x^2$



2. $y = -\frac{1}{6}x^2$



3. $x = -\frac{1}{3}y^2$



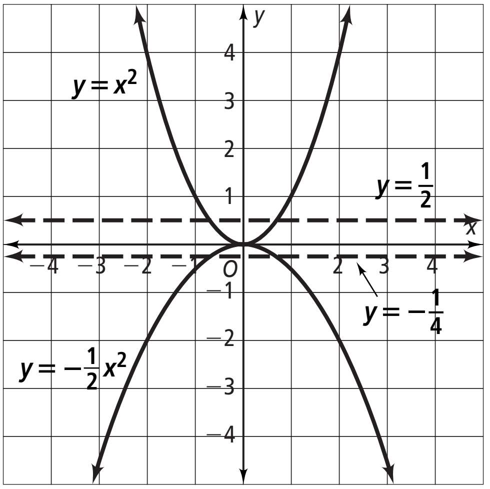
Reteaching (continued)

A New Look at Parabolas

You can analyze the equation of a parabola by analyzing transformations of the parent function $y = x^2$.

Problem

What are the vertices, foci, and directrices of the parabolas $y = x^2$ and $y = -\frac{1}{2}x^2$?



The equation $y = -\frac{1}{2}x^2$ is a transformation of $y = x^2$.

Both parabolas have vertices at (0, 0).

Both equations can be written in the form $y = \frac{1}{4c}x^2$.

The focus is $(0, c)$, and the directrix is $y = -c$.
To find the focus $(0, c)$ of $y = x^2$, find the value of c.

You know that $y = x^2 = \frac{1}{1}x^2 = \frac{1}{4 \cdot \frac{1}{4}}x^2$ so $c = \frac{1}{4}$.

The focus is $\left(0, \frac{1}{4}\right)$ and the directrix is $y = -\frac{1}{4}$.
In the transformed equation $y = -\frac{1}{2}x^2$, you know that the equation opens downward. To find the focus

$(0, c)$ of $y = -\frac{1}{2}x^2$, identify the value of c: $y = -\frac{1}{2}x^2 = \frac{1}{4\left(-\frac{1}{2}\right)}x^2$.

The focus is $\left(0, -\frac{1}{2}\right)$ and the directrix is $y = \frac{1}{2}$.

Exercises

Identify the vertex, focus, and directrix of the parabola.

4. $y = 2x^2$ **(0, 0); $\left(0, \frac{1}{8}\right)$; $y = -\frac{1}{8}$**

5. $y = -3x^2$ **(0, 0); $\left(0, -\frac{1}{12}\right)$; $y = \frac{1}{12}$**

6. $y = \frac{1}{8}x^2$ **(0, 0); (0, 2); $y = -2$**

7. $y = \frac{1}{4}x^2$ **(0, 0); (0, 1); $y = -1$**

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Reteaching

Circles in the Coordinate Plane

- When working with circles, begin by writing the equation in standard form:
$$(x - h)^2 + (y - k)^2 = r^2$$
- Unlike equations of parabolas, which include either x^2 or y^2, the equation of a circle will include both x^2 and y^2.

Problem

What is the center and radius of the circle with the equation $x^2 + y^2 - 4x + 6y = 12$?

$(x^2 - 4x)$	$+$	$(y^2 + 6y) = 12$	Rearrange and group the terms by variable.
$\frac{b}{2} = \frac{-4}{2} = -2$		$\frac{b}{2} = \frac{6}{2} = 3$	To complete each square, find $\frac{b}{2}$ for each group.
$\left(\frac{b}{2}\right)^2 = (-2)^2 = 4$		$\left(\frac{b}{2}\right)^2 = (3)^2 = 9$	Find $\left(\frac{b}{2}\right)^2$ for each expression.
$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$			Add $\left(\frac{b}{2}\right)^2$ for each expression to both sides.
$(x - 2)^2 + (y + 3)^2 = 25$			Write the expressions as perfect squares; simplify.
$(x - 2)^2 + (y - (-3))^2 = 5^2$			Write the equation in standard form.
$h = 2, k = -3, r = 5$			Compare the equation to $(x - h)^2 + (y - k)^2 = r^2$.

The center of the circle is $(2, -3)$. The radius of the circle is 5.

Exercises

Find the center and radius of each circle.

1. $x^2 + y^2 - 10y = 0$ **(0, 5); 5**
2. $x^2 + y^2 = 225$ **(0, 0); 15**
3. $x^2 + y^2 + 2x - 6y = 15$ **(−1, 3); 5**
4. $x^2 + y^2 + 12x + 14y = -84$ **(−6, −7); 1**
5. $x^2 + y^2 + 2x + 4y = 31$ **(−1, −2); 6**
6. $x^2 + y^2 - 10x - 4y = -20$ **(5, 2); 3**
7. $x^2 + y^2 + 16x - 8y = -72$ **(−8, 4); $2\sqrt{2}$**
8. $x^2 + y^2 - 8x + 6y = -5$ **(4, −3); $2\sqrt{5}$**
9. $x^2 + y^2 - 4x - 6y = -4$ **(2, 3); 3**
10. $x^2 + y^2 + 8x = 47$ **(−4, 0); $3\sqrt{7}$**

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Reteaching (continued)

Circles in the Coordinate Plane

Problem

In order to plan a circular ornamental garden, a landscaper uses grid paper in which the center of the garden is at (0, 8) and the diameter is 20 units. How can he model the circular garden with an equation?

Step 1 Identify the information that you know.
The center of the circle is (0, 8). The diameter of the circle is 20.

Step 2 Identify the information that you want to find.
You need to find h, k, and r. The diameter of the circle is 20 units, so the radius is $\frac{20}{2} = 10$ units. The center is (0, 8), so $h = 0$ and $k = 8$.

Step 3 Substitute known values into the standard form of an equation for a circle.

$(x - h)^2 + (y - k)^2 = r^2$	Write the standard form.
$(x - 0)^2 + (y - 8)^2 = 10^2$	Substitute 0 for h, 8 for k, and 10 for r.
$x^2 + (y - 8)^2 = 100$	Simplify.

Exercises

11. An archaeologist is working on the excavation of a circular shaped pit. If the center of the pit is (3, 7) and the radius is 30 ft, what is an equation for the circle?

$(x - 3)^2 + (y - 7)^2 = 900$

12. A homeowner is planning a circular sandbox in the backyard. She wants the diameter of the sandbox to be 15 ft. She uses graph paper and marks the center of the circle at (−2, −5). What is an equation for the circle?

$(x + 2)^2 + (y + 5)^2 = 56.25$

13. A quilter is cutting pieces for a quilt with a circular design in the center. If the center of the circle is the origin and the radius of the circle is 12 in., what is an equation for the circle?

$x^2 + y^2 = 144$

14. The diameter of a playground merry-go-round is 6 ft. If it is placed on a grid with the center at (−9, 7), what equation models it?

$(x + 9)^2 + (y - 7)^2 = 9$