Name ______________________ Class __________ Date __________

Reteaching

Solving Equations

To solve an equation that contains a variable, find all of the values of the variable that make the equation true. Use the equality properties of real numbers and inverse operations to rewrite the equation until the variable is alone on one side of the equation. Whatever remains on the other side of the equation is the solution.

Addition Property of Equality	*Subtraction Property of Equality*
To isolate $\frac{w}{2}$ on one side of the equation, *add* 7 to each side. $\frac{w}{2} - 7 = 11$; $+7 \quad +7$; $\frac{w}{2} = 18$	To isolate 5z on one side of the equation, *subtract* 3 from each side. $5z + 3 = 13$; $-3 \quad -3$; $5z = 10$
Multiplication Property of Equality	***Division Property of Equality***
To isolate w on one side of the equation, *multiply* each side by 2. $\frac{w}{2} = 18$; $\times 2 \quad \times 2$; $w = 36$	To isolate z on one side of the equation, *divide* each side by 5. $\frac{5z}{5} = \frac{10}{5}$; $z = 2$

Exercises

Solve each equation.

1. $y + 12 = 8$ **−4**

2. $p - 9 = 12$ **21**

3. $-3 + r = 20$ **23**

4. $8 = 15 + k$ **−7**

5. $9q = 27$ **3**

6. $\frac{t}{6} = -4$ **−24**

7. $5 = -\frac{d}{7}$ **−35**

8. $49 = 10m$ **4.9**

9. $3g - 14 = 13$ **9**

10. $\frac{n}{8} + 19 = 3$ **−128**

11. $27 = -5 - \frac{c}{4}$ **−128**

12. $12 = 2f + 9$ **1.5**

Name ______________________________ Class ____________ Date ____________

Reteaching (continued)

Solving Equations

To solve an equation for one of its variables, rewrite the equation as an equivalent equation with the specified variable on one side of the equation by itself and an expression not containing that variable on the other side.

Problem

The equation $\frac{ax - b}{2} = x + 2b$ defines a relationship between a, b, and x. What is x in terms of a and b?

Use the properties of equality and the properties of real numbers to rewrite the equation as a sequence of equivalent equations.

$\frac{ax - b}{2} = x + 2b$	
$2\left(\frac{ax - b}{2}\right) = 2(x + 2b)$	Multiply each side by 2.
$ax - b = 2(x + 2b)$	Simplify.
$ax - b = 2x + 4b$	Distributive Property
$ax - 2x = 4b + b$	Add and subtract to get terms with x on one side and terms without x on the other side.
$ax - 2x = 5b$	Simplify.
$x(a - 2) = 5b$	Distributive Property
$x = \frac{5b}{a - 2}$	Divide each side by $a - 2$.

The final form of the equation has x on the left side by itself and an expression not containing x on the right side.

Exercises

Solve each equation for the indicated variable.

13. $3m - n = 2m + n$, for m $m = 2n$

14. $2(u + 3v) = w - 5u$, for u $u = \frac{w - 6v}{7}$

15. $ax + b = cx + d$, for x $x = \frac{d - b}{a - c}$

16. $k(y + 3z) = 4(y - 5)$, for y $y = -\frac{3kz + 20}{k - 4}$

17. $\frac{1}{2}r + 3s = 1$, for r $r = 2 - 6s$

18. $\frac{2}{3}f + \frac{5}{12}g = 1 - fg$, for f $f = \frac{12 - 5g}{8 + 12g}$

19. $\frac{x + k}{j} = \frac{3}{4}$, for x $x = \frac{3j - 4k}{4}$

20. $\frac{a - 3y}{b} + 4 = a + y$, for y $y = \frac{a + 4b - ab}{b + 3}$

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Reteaching

Solving Inequalities

As with an equation, the solutions of an inequality are numbers that make it true. The procedure for solving a linear inequality is much like the one for solving linear equations. To isolate the variable on one side of the inequality, perform the same algebraic operation on each side of the inequality symbol.

The **Addition and Subtraction Properties of Inequality** state that adding or subtracting the same number from both sides of the inequality does not change the inequality.

If $a < b$, then $a + c < b + c$. If $a < b$, then $a - c < b - c$.

The **Multiplication and Division Properties of Inequality** state that multiplying or dividing both sides of the inequality by the same *positive* number does not change the inequality.

If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

Problem

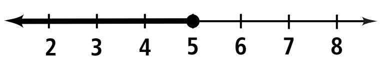
What is the solution of $3(x + 2) - 5 \leq 21 - x$? Graph the solution.

Justify each line in the solution by naming one of the properties of inequalities.

$3x + 6 - 5 \leq 21 - x$	Distributive Property
$3x + 1 \leq 21 - x$	Simplify.
$4x + 1 \leq 21$	Addition Property of Inequality
$4x \leq 20$	Subtraction Property of Inequality
$x \leq 5$	Division Property of Inequality

To graph the solution, locate the boundary point. Plot a point at $x = 5$. Because the inequality is "*less than or equal to*," the boundary point is part of the solution set. Therefore, use a closed dot to graph the boundary point. Shade the number line to the left of the boundary point because the inequality is "*less than*."

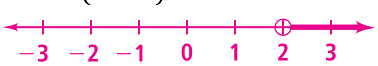
Graph the solution on a number line.



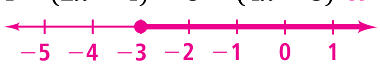
Exercises

Solve each inequality. Graph the solution.

1. $2x + 4(x - 2) > 4$ $x > 2$



2. $4 - (2x - 4) \geq 5 - (4x + 3)$ $x \geq -3$



Reteaching (continued)

Solving Inequalities

The procedure for solving an inequality is similar to the procedure for solving an equation but with one important exception.

The Multiplication and Division Properties of Equality also state that, when you multiply or divide each side of an inequality by a negative number, you must reverse the inequality symbol.

If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

Problem

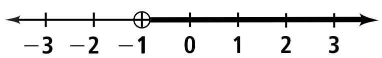
What is the solution of $2x - 3(x - 1) < x + 5$? Graph the solution.

Justify each line in the solution by naming one of the properties of inequalities.

$2x - 3(x - 1) < x + 5$	
$2x - 3x + 3 < x + 5$	Distributive Property
$-x + 3 < x + 5$	Simplify.
$-2x < 2$	Subtraction Property of Inequality
$x > -1$	Division Property of Inequality

The direction of the inequality changed in the last step because we divided both sides of the inequality by a negative number.

Graph the solution on a number line.



Exercises

Solve each inequality.

3. $x - 1 \leq -4(-2 - x)$ **$x \geq -3$**

4. $7 - 7(x - 7) > -4 + 5x$ **$x < 5$**

5. $7(x + 4) - 13 \geq 12 + 13(3 + x)$ **$x \leq -6$**

6. $4x - 1 < 6x - 5$ **$x > 2$**

Name ______________________ Class ____________ Date ____________

Reteaching

Linear Functions and Slope-Intercept Form

You can use the slope-intercept form to write equations of lines.

- The slope-intercept formula is $y = mx + b$, where m represents the slope of the line, and b represents its y-intercept. The y-intercept is the point at which the line crosses the y-axis.
- The slope of a horizontal line is always zero, and the slope of a vertical line is always undefined.

Problem

What is the equation of the line that contains the point $(3, -1)$ and has a slope of $-\frac{4}{3}$?

$-1 = \left(-\frac{4}{3}\right)(3) + b$	To find b, substitute the values $-\frac{4}{3}$ for m, 3 for x, and -1 for y, into the slope-intercept formula.
$-1 = -4 + b$	Multiply.
$3 = b$	Add 4 to each side and simplify.
$y = -\frac{4}{3}x + 3$	Substitute $-\frac{4}{3}$ for m and 3 for b into the slope-intercept formula.

Exercises

Write an equation for each line.

1. $m = 4$; contains $(3, 2)$
$y = 4x - 10$

2. $m = -2$; contains $(4, 7)$
$y = -2x + 15$

3. $m = 0$; contains $(3, 0)$
$y = 0$

4. $m = -1$; contains $(-5, -2)$
$y = -x - 7$

5. $m = 3$; contains $(-2, -4)$
$y = 3x + 2$

6. $m = 0$; contains $(0, -7)$
$y = -7$

7. $m = 8$; contains $(5, 0)$
$y = 8x - 40$

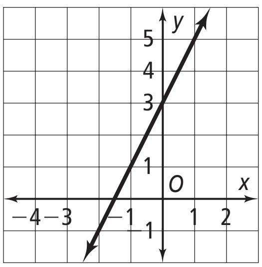
8. $m = -1$; contains $(0, 7)$
$y = -x + 7$

9. $m = 0$; contains $(3, 8)$
$y = 8$

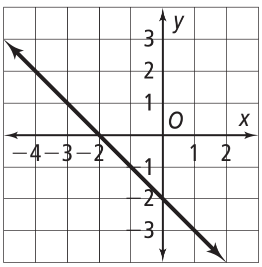
10. $m = 4$; contains $(2, 5)$
$y = 4x - 3$

11. $m = 7$; contains $(3, 2)$
$y = 7x - 19$

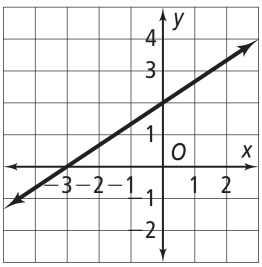
12. $m = -1$; contains $(2, -6)$
$y = -x - 4$

13.


$y = 2x + 3$

14.


$y = -x - 2$

15.


$y = \frac{2}{3}x + 2$

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Reteaching (continued)

Linear Functions and Slope-Intercept Form

You can graph a linear equation if you know the slope and the y-intercept.

- Write the linear equation in slope-intercept form.
- Plot the y-intercept.
- Plot a second point using the slope.
- Draw a line through the two points.

Problem

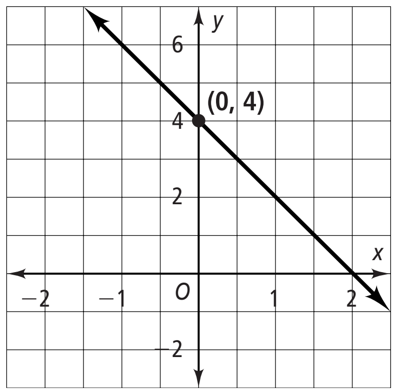
What is the graph of $4x + 2y = 8$?

Write the linear equation in slope-intercept form.

$4x + 2y = 8$	Original equation
$2y = -4x + 8$	
$y = -2x + 4$	Slope-intercept form

The slope is -2 and the y-intercept is $(0, 4)$.

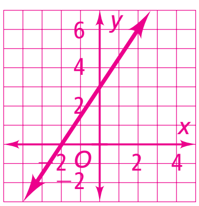
Plot the y-intercept. Use the slope to plot a second point. Then draw a line through the two points.



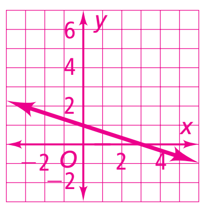
Exercises

Graph each equation.

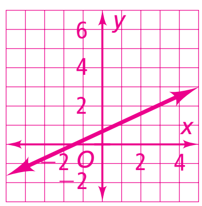
16. $-3x + 2y = 6$



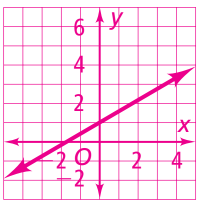
17. $3y + x = 3$



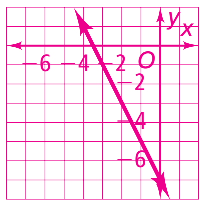
18. $3y - x = 2$



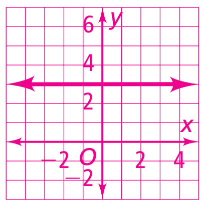
19. $-2x + 4y - 3 = 0$



20. $y + 7 = -2x$



21. $2y - 6 = 0$



Name ______________________________ Class ____________ Date ____________

Reteaching

Families of Functions

Horizontal and Vertical Translations

If h and k are positive numbers, then

$g(x) = f(x) + k$ shifts the graph of $f(x)$ **up** k units.

$g(x) = f(x) - k$ shifts the graph of $f(x)$ **down** k units.

$g(x) = f(x + h)$ shifts the graph of $f(x)$ **left** h units.

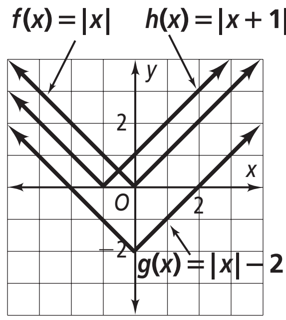
$g(x) = f(x + h)$ shifts the graph of $f(x)$ **right** h units.

Problem

How can you represent each translation of $y = |x|$ graphically?

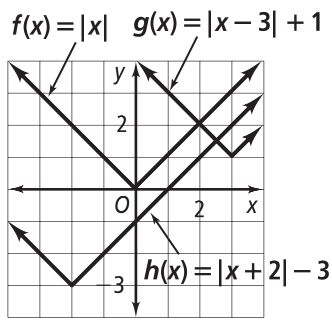
1. a. $g(x) = |x| - 2$ Shift the graph of $f(x) = |x|$ down 2 units.

b. $h(x) = |x + 1|$ Shift the graph of $f(x) = |x|$ left 1 unit.



2. a. $g(x) = |x - 3| + 1$ Shift the graph of $f(x) = |x|$ right 3 units and up 1 unit.

b. $h(x) = |x + 2| - 3$ Shift the graph of $f(x) = |x|$ left 2 units and down 3 units.



Exercises

Identify the type of translation of $f(x) = |x|$.

1. $g(x) = |x - 2|$ right 2 units

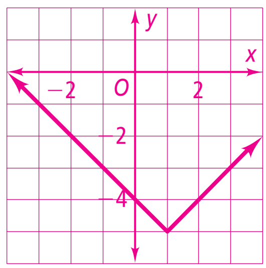
2. $g(x) = |x| + 1$ up 1 unit

3. $g(x) = |x| - 3$ down 3 units

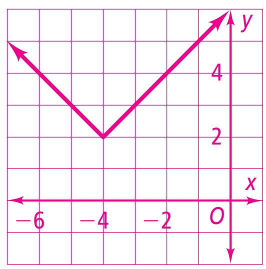
4. $g(x) = |x + 3|$ left 3 units

Graph each translation of $f(x) = |x|$.

5. $g(x) = |x - 1| - 5$



6. $g(x) = |x + 4| + 2$



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Reteaching (continued)

Families of Functions

Reflection, Stretching, and Compression

If h and k are positive numbers, then

$g(x) = -f(x)$ reflects the graph of $f(x)$ in the x–axis.

$g(x) = f(-x)$ reflects the graph of $f(x)$ in the **y–axis**.

$g(x) = af(x)$, $a > 1$, is a vertical **stretch** of the graph of $f(x)$.

$g(x) = af(x)$, $0 < a < 1$, is a vertical **compression** the graph of $f(x)$.

Problem

What transformations change the graph of $f(x)$ to $g(x)$?

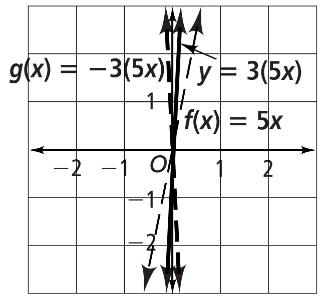
$f(x) = 5x$ $\qquad g(x) = -3(5x)$

There are two transformations.

First transformation: The graph of $y = 3(5x)$ is the graph of $f(x) = 5x$ stretched vertically by a factor of 3 because $a = 3$ and $a > 1$.

Second transformation: The graph of $g(x) = -3(5x)$ is the graph of $y = 3f(5x)$ reflected in the x-axis because the sign of $g(x)$ has changed.

So, the graph of $g(x)$ is the graph of $f(x)$ stretched vertically by a factor of 3 and reflected over the x-axis.



Exercises

Describe the transformations of $f(x)$ that produce $g(x)$.

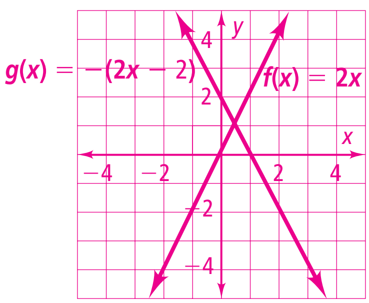
7. $f(x) = -5x$, $g(x) = x$ — vertically compressed by a factor of $\frac{1}{5}$ and reflected in the x-axis

8. $f(x) = x$, $g(x) = \frac{1}{4}x + 3$ — vertically compressed by a factor of $\frac{1}{4}$ and translated up 3 units

Graph $f(x)$ and $g(x)$ on the same coordinate plane.

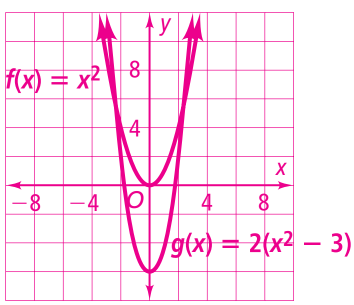
9. $f(x) = 2x$

$g(x) = -(2x - 2)$



10. $f(x) = x^2$

$g(x) = 2(x^2 - 3)$



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Reteaching

Absolute Value Functions and Graphs

A function of the form $y = a|x - h| + k$ is an *absolute value function*. The graph of $y = a|x - h| + k$ is an angle; its vertex is located at the point (h, k).

Problem

What is the graph of $y = 2|x + 3| - 1$?

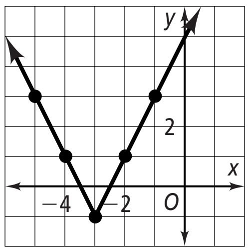
This function is in the general form $y = a|x - h| + k$ where $a = 2$, $h = -3$, and $k = -1$. The vertex is (h, k) or $(-3, -1)$.

Now make a table showing several points on the graph. Choose values of x on both sides of the vertex.

x	−5	−4	−3	−2	−1
y	3	1	−1	1	3

Plot the vertex and the points from the table.

Connect the points to graph the function.

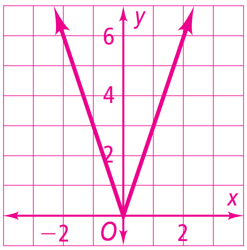


Exercises

Make a table of values for each equation. Then graph the equation.

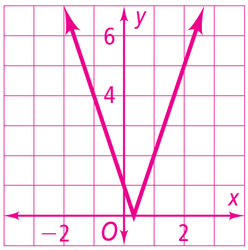
1. $y = |3x|$

Table of values may vary.



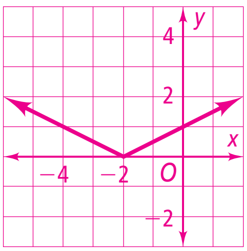
2. $y = |3x - 1|$

Table of values may vary.



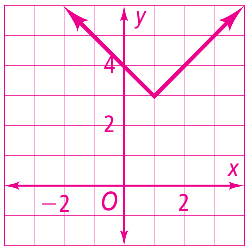
3. $y = \left|\frac{1}{2}x + 1\right|$

Table of values may vary.



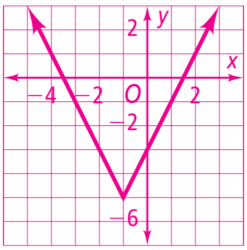
4. $y = |x - 1| + 3$

Table of values may vary.



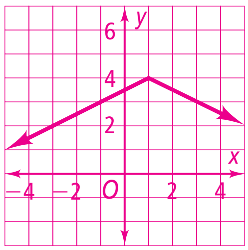
5. $y = 2|x + 1| - 5$

Table of values may vary.



6. $y = -\frac{1}{2}|x - 1| + 4$

Table of values may vary.



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Reteaching (continued)

Absolute Value Functions and Graphs

The table below shows the transformations of the graph of the parent function $y = |x|$. A translation is a transformation that shifts a graph vertically or horizontally without changing the graph's shape, size, or orientation.

Vertical Translation of $y = \|x\|$ $y = \|x\| + k$ If $k > 0$, translate UP k units. If $k < 0$, translate DOWN $\|k\|$ units.	**Horizontal Translation of $y = \|x\|$** $y = \|x - h\|$ If $h > 0$, translate to the RIGHT h units. If $h < 0$, translate to the LEFT $\|h\|$ units.
Vertical Stretch and Compression of $y = \|x\|$ $y = a\|x\|, a > 0$ If $a > 1$, the graph is narrower. This is a vertical stretch of the parent graph. If $a < 1$, the graph is wider. This is a vertical compression of the parent graph.	**Reflection of $y = \|x\|$** $y = -\|x\|$ The graph of the parent function is reflected in the x-axis.

Problem

Compare $y = -3|x + 1| + 2$ with the parent function $f(x) = |x|$. Without graphing, find the vertex, axis of symmetry, and transformations.

First find the vertex:

This function is in the general form $y = a|x - h| + k$ where $a = -3$, $h = -1$, and k=2. The vertex is $(-1, 2)$. The axis of symmetry is $x = -1$.

Describe the transformation of the parent function $f(x) = |x|$:

$k = 2$, so the graph is translated up 2 units.

$a = -3$, so the graph is reflected in the x-axis AND vertically stretched by a factor of 3.

$h = -1$, so the graph is translated to the left 1 unit.

Exercises

Without graphing, identify the vertex, axis of symmetry, and transformations from the parent function $f(x) = |x|$.

7. $y = |x - 1| + 2$
   (1, 2); $x = 1$; translated 1 unit right and 2 units up

8. $y = 3|x|$
   (0, 0); $x = 0$; vertical stretch by a factor of 3

9. $y = 2|x + 1| - 3$
   $(-1, -3)$; $x = -1$; translated 1 unit left, vertically stretched by a factor of 2, 3 units down

10. $y = -\frac{1}{2}|x|$
    (0, 0); $x = 0$; reflected in the x-axis, vertically compressed by a factor of $\frac{1}{2}$

11. $y = \frac{3}{2}|x| + 2$
    (0, 2); $x = 0$; vertically stretched by a factor of $\frac{3}{2}$, translated 2 units up

12. $y = 4|x - 5| + 3$
    (5, 3); $x = 5$; translated 5 units right, vertically stretched by a factor of 4, 3 units up

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Reteaching

Two-Variable Inequalities

A **linear inequality** in two variables is an inequality whose graph is a region of the coordinate plane bounded by a line. This line is the **boundary**. If the boundary is included in the solution of the inequality, it is drawn as a solid line. If the boundary is not part of the solution of the inequality, it is drawn as a dashed line.

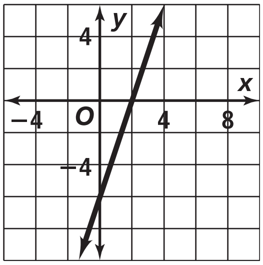
Problem

What is the graph of $6x - 2y \leq 12$?

$6x - 2y \leq 12$

$y \geq 3x - 6$

To graph the boundary line, write the inequality in slope-intercept form as if it were an equation.



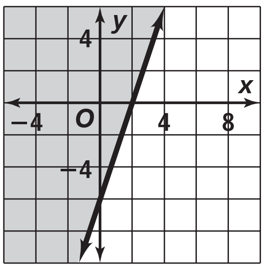
The boundary line is solid if the inequality contains $\leq$ or $\geq$. The boundary line is dashed if the inequality contains $<$ or $>$. Graph the boundary line $y = 3x - 6$ as a solid line.

$0 \geq 3(0) - 6$

Since the boundary line does not contain the origin, substitute the point (0, 0) into the inequality.

$0 \geq -6$

Simplify. The resulting inequality is true.

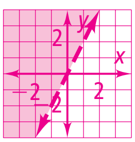


Shade the region that contains the origin. If the resulting inequality were false, then you would shade the region that does not contain the origin.

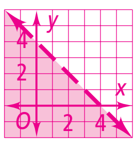
Exercises

Graph each inequality.

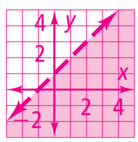
1. $y > 2x$



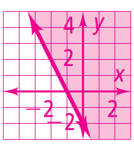
2. $x + y < 4$



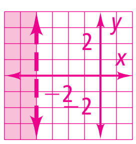
3. $y < x + 1$



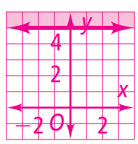
4. $3x - 2 \leq 5x + y$



5. $x < -4$



6. $y \geq 5$



Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Two-Variable Inequalities

To graph two-variable absolute value inequalities, graph the boundary line. Then pick a test point and shade appropriately.

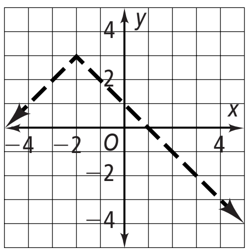
Problem

What is the graph of $3 - y < |x + 2|$?

$3 - y < |x + 2|$

$y > -|x + 2| + 3$

To graph the boundary line, write the inequality in terms of y as if it were an equation. The boundary line is dashed because the inequality contains $>$.



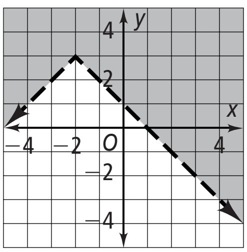
Graph the boundary line $y = -|x + 2| + 3$.

$0 \geq -|0 + 2| + 3$

Now pick a test point. Because the boundary line does not contain the origin, substitute the point (0, 0) into the inequality.

$0 \not\geq 1$

Simplify. The resulting inequality is untrue.

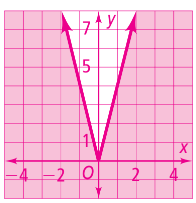


Shade the region that *does not* contain the origin. If the resulting inequality were true, then you would shade the region that *does* contain the origin.

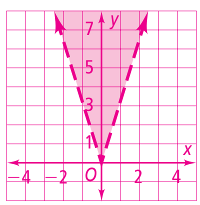
Exercises

Graph each absolute value inequality.

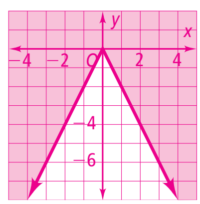
7. $y \leq |4x|$



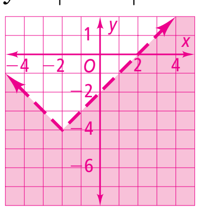
8. $y > |-3x|$



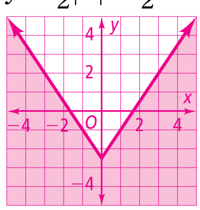
9. $y \geq -|2x|$



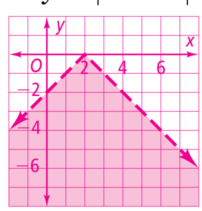
10. $y < |x + 2| - 4$



11. $y \leq \frac{3}{2}|x| - \frac{5}{2}$



12. $-3y > |3x - 6|$



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Reteaching

Solving Systems Algebraically

Follow these steps when solving by substitution.

Step 1 Solve one equation for one of the variables.

Step 2 Substitute the expression for this first variable into the other equation. Solve for the second variable.

Step 3 Substitute the second variable's value into either equation. Solve for the first variable.

Step 4 Check the solution in the other original equation.

Problem

What is the solution of the system of equations? $\begin{cases} 4x + 3y = 10 \\ x + 2y = 10 \end{cases}$

Step 1 $x = -2y + 10$ — Solve one equation for x.

Step 2

$4(-2y + 10) + 3y = 10$	Substitute the expression for x into the other equation.
$-8y + 40 + 3y = 10$	Distribute.
$-5y = -30$	Combine like terms.
$y = 6$	Solve for y.

Step 3

$x + 2(6) = 10$	Substitute the y value into either equation.
$x + 12 = 10$	Simplify.
$x = -2$	Solve for x.

Step 4

$4(-2) + 3(6) \stackrel{?}{=} 10$	Check the solution in the other equation.
$-8 + 18 \stackrel{?}{=} 10$	Simplify.
$10 = 10$ ✓	

The solution is $(-2, 6)$.

Exercises

Solve each system by substitution.

1. $\begin{cases} x - 3y = 2 \\ -x + 2y = 5 \end{cases}$ **$x = -19, y = -7$**

2. $\begin{cases} a + 3b = 4 \\ a = -2 \end{cases}$ **$a = -2, b = 2$**

3. $\begin{cases} -2m + n = 6 \\ -7m + 6n = 1 \end{cases}$ **$m = -7, n = -8$**

4. $\begin{cases} 7x - 3y = -1 \\ x + 2y = 12 \end{cases}$ **$x = 2, y = 5$**

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Solving Systems Algebraically

Follow these steps when solving by elimination.

Step 1 Arrange the equations with like terms in columns. Circle the like terms for which you want to obtain coefficients that are opposites.

Step 2 Multiply each term of one or both equations by an appropriate number.

Step 3 Add the equations.

Step 4 Solve for the remaining variable.

Step 5 Substitute the value obtained in step 4 into either of the original equations, and solve for the other variable.

Step 6 Check the solution in the other original equation.

Problem

What is the solution of the system of equations? $\begin{cases} 2x + 5y = 11 \\ 3x - 2y = -12 \end{cases}$

Step 1 $\textcircled{2x} + 5y = 11$, $\textcircled{3x} - 2y = -12$ Circle the terms that you want to make opposite.

Step 2 $6x + 15y = 33$ Multiply each term of the first equation by 3.
$-6x + 4y = 24$ Multiply each term of the second equation by -2.

Step 3 $19y = 57$ Add the equations.

Step 4 $y = 3$ Solve for the remaining variable.

Step 5 $3x - 2(3) = -12$, $x = -2$ Substitute 3 for *y* to solve for *x*.

Step 6 Check using the other equation.

$$2(-2) + 5(3) \stackrel{?}{=} 11$$
$$-4 + 15 \stackrel{?}{=} 11$$
$$11 = 11 \checkmark$$

The solution is $(-2, 3)$. You can also check the solution by using a graphing calculator.

Exercises

Solve each system by elimination.

5. $\begin{cases} 3x + 2y = -17 \\ x - 3y = 9 \end{cases}$ **$x = -3, y = -4$**

6. $\begin{cases} 5f + 4m = 6 \\ -2f - 3m = -1 \end{cases}$ **$f = 2, m = -1$**

7. $\begin{cases} 3x - 2y = 5 \\ -6x + 4y = 7 \end{cases}$ **no solution**

8. $\begin{cases} -2x - 4y = 2 \\ 10x + 20y = -10 \end{cases}$ **$y = -\frac{1}{2}x - \frac{1}{2}$, where x is any real number**

9. **Reasoning** Why does a system with no solution represent parallel lines?
If there is no solution, then there are no values of the variables that will make both equations true. This means there is no point that lies on both lines, so the lines never meet and are therefore parallel.

Name ______________________ Class ____________ Date ____________

Reteaching

Systems of Inequalities

Solving a System by Using a Table

Problem

An English class has 4 computers for at most 18 students. Students can either use the computers in groups to research Shakespeare or to watch an online performance of Macbeth. Each research group must have 4 students and each performance group must have 5 students. In how many ways can you set up the computer groups?

Step 1 Relate the unknowns and define them with variables.

x = number of research groups, y = number of performance groups

number of research groups + number of performance groups $\leq$ 4

4 • number of research groups + 5 • number of performance groups $\leq$ 18

$$x + y \leq 4$$
$$4x + 5y \leq 18$$

Step 2 Make a table of values for x and y that satisfy the first inequality. The replacement values for x and y must be whole numbers.

x	y
0	4, 3, 2, 1, 0
1	3, 2, 1, 0
2	2, 1, 0
3	1, 0
4	0

Step 3 In the table, check each pair of values to see which satisfy the other inequality. Highlight these pairs. These are the solutions of the system.

x	y
0	4, 3, 2, 1, 0
1	3, 2, 1, 0
2	2, 1, 0
3	1, 0
4	0

You can have:
0 groups doing research and 0, 1, 2, or 3 groups watching performances or
1 group doing research and 0, 1, or 2 groups watching performances or
2 groups doing research and 0, 1, or 2 groups watching performances or
3 groups doing research and 0 or 1 group watching performances or
4 groups doing research and 0 groups watching performances

Exercises

Find the whole number solutions of each system using tables.

1. $\begin{cases} x + y < 4 \\ x + 2y \leq 10 \end{cases}$

(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (3, 0)

2. $\begin{cases} x - y \geq 1 \\ 6x + 3y \leq 21 \end{cases}$

(1, 0), (2, 0), (2, 1), (3, 0), (3, 1)

3. $\begin{cases} x + y \geq 5 \\ y < -2x + 8 \end{cases}$

(0, 5), (0, 6), (0, 7), (1, 4), (1, 5), (2, 3)

4. $\begin{cases} y < 3 \\ 4x + 2y < 12 \end{cases}$

(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1)

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Systems of Inequalities

Solving a System by Graphing

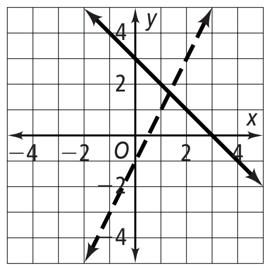
Problem

What is the solution of the system of inequalities? $\begin{cases} 2x - y > 1 \\ x + y \geq 3 \end{cases}$

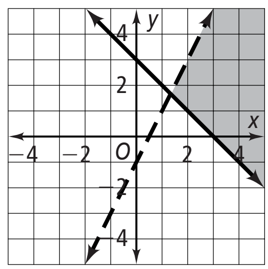
Step 1 Solve each inequality for *y*.

$$2x - y > 1 \qquad\qquad x + y \geq 3$$
$$-y > -2x + 1 \quad \text{and} \quad y \geq -x + 3$$
$$y < 2x - 1$$

Step 2 Graph the boundary lines. Use a solid line for $\geq$ or $\leq$ inequalities. Use a dotted line for $>$ and $<$ inequalities.



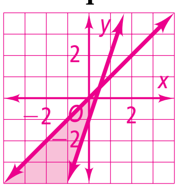
Step 3 Shade on the appropriate side of each boundary line. The overlap is the solution to the system.



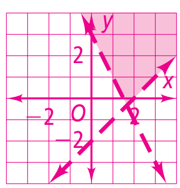
Exercises

Solve each system of inequalities by graphing.

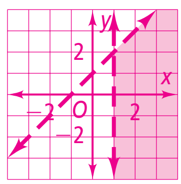
5. $\begin{cases} y \leq x \\ y \geq 3x - 1 \end{cases}$



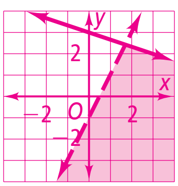
6. $\begin{cases} 2x + y > 3 \\ x - y < 2 \end{cases}$



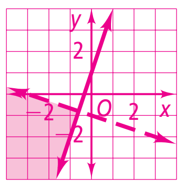
7. $\begin{cases} x > 1 \\ y < x + 1 \end{cases}$



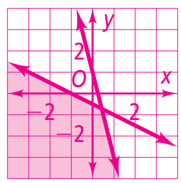
8. $\begin{cases} x + 3y \leq 9 \\ 2x - \ \ y > 1 \end{cases}$



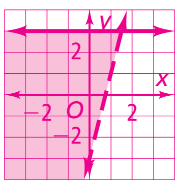
9. $\begin{cases} y < -\frac{1}{3}x - 1 \\ y \geq 3x + 1 \end{cases}$



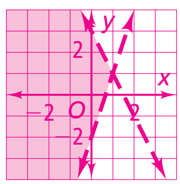
10. $\begin{cases} 4x + \ \ y \leq 1 \\ x + 2y \leq -1 \end{cases}$



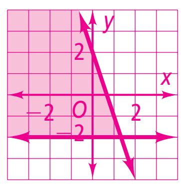
11. $\begin{cases} y \leq 3 \\ y > 4x - 3 \end{cases}$



12. $\begin{cases} 2x + y < 3 \\ 3x - y < 2 \end{cases}$



13. $\begin{cases} y \geq -2 \\ 3x + y \leq 2 \end{cases}$



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Reteaching

Systems With Three Variables

Problem

What is the solution of the system?

$$\begin{cases} x + y + z = 6 & (1) \\ 2x - y + 3z = 9 & (2) \\ -x + 2y + 2z = 9 & (3) \end{cases}$$

Use elimination. The equations are numbered to make the process easy to follow.

$$\begin{aligned} (1)\quad x + y + z &= 6 \\ (3)\quad -x + 2y + 2z &= 9 \\ \hline (4)\quad 3y + 3z &= 15 \end{aligned}$$

Pair the equations to eliminate x.

$$\begin{aligned} (2)\quad 2x - y + 3z &= 9 \\ (3)\quad -x + 2y + 2z &= 9 \end{aligned}$$

Pair a different set of equations.

$$\begin{aligned} (2)\quad 2x - y + 3z &= 9 \\ (3)\quad -2x + 4y + 4z &= 18 \\ \hline (5)\quad 3y + 7z &= 27 \end{aligned}$$

Multiply equation 3 by 2 to eliminate x. Then add the two equations.

$$\begin{aligned} (4)\quad 3y + 3z &= 15 \\ (5)\quad -3y + (-7z) &= -27 \\ \hline -4z &= -12 \\ z &= 3 \end{aligned}$$

Equations 4 and 5 form a system. Multiply equation 5 by -1 and add to equation 4 to eliminate y and solve for z.

$$\begin{aligned} (4)\quad 3y + 3(3) &= 15 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

Substitute $z = 3$ into equation 4 and solve for y.

$$\begin{aligned} (1)\quad x + 2 + 3 &= 6 \\ x &= 1 \end{aligned}$$

Substitute the values of y and z into one of the original equations. Solve for x.

The solution is (1, 2, 3).

Exercises

Solve each system by elimination. Check your answers.

1. $\begin{cases} 2x - y + 2z = 10 \\ 4x + 2y - 5z = 10 \\ x - 3y + 5z = 8 \end{cases}$

(4, 2, 2)

2. $\begin{cases} x - y + z = 6 \\ 2x + 3y + 2z = 2 \\ 3x + 5y + 4z = 4 \end{cases}$

(2, −2, 2)

3. $\begin{cases} 6x - 4y + 5z = 31 \\ 5x + 2y + 2z = 13 \\ x + y + z = 2 \end{cases}$

(3, −2, 1)

4. $\begin{cases} 3x + y + z = 2 \\ 4x - 2y + 3z = -4 \\ 2x + 2y + 2z = 8 \end{cases}$

(−1, 3, 2)

5. $\begin{cases} 5x + 2y + z = 5 \\ 3x - 3y - 3z = 9 \\ x + 2y + 4z = 6 \end{cases}$

(2, −4, 3)

6. $\begin{cases} x + y + z = -1 \\ 4x + 3y + 2z = -10 \\ 2x - 4y - 2z = -6 \end{cases}$

(−3, −2, 4)

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Systems With Three Variables

Problem

What is the solution of the system? Use substitution.

$$\begin{cases} ① & x + 3y - 2z = 19 \\ ② & 4x - 2y + 3z = 8 \\ ③ & -3x + 2y + 2z = 15 \end{cases}$$

Step 1 Choose an equation that can be solved easily for one variable. Choose equation 1 and solve for x.

① $x + 3y - 2z = 19$

$x = -3y + 2z + 19$

Step 2 Substitute the expression for x into equations 2 and 3 and simplify.

$$\begin{aligned} 4(-3y + 2z + 19) - 2y + 3z &= 8 \\ -12y + 8z + 76 - 2y + 3z &= 8 \\ ④ \quad -14y + 11z &= -68 \end{aligned}$$

$$\begin{aligned} -3(-3y + 2z + 19) + 2y + 2z &= 15 \\ 9y - 6z - 57 + 2y + 2z &= 15 \\ ⑤ \quad 11y - 4z &= 72 \end{aligned}$$

Step 3 Write the two new equations as a system. Solve for y and z.

$$\begin{cases} ④ & -14y + 11z = -68 \\ ⑤ & 11y - 4z = 72 \end{cases}$$

$-56y + 44z = -272$ Multiply ④ by 4.

$121y - 44z = 792$ Multiply ⑤ by 11.

$65y = 520$

$y = 8$

$-14y + 11z = -68$ Substitute $y = 8$ into ④.

$-14(8) + 11z = -68$

$11z = 44$

$z = 4$

Step 4 Use one of the original equations to solve for x.

$x + 3y - 2z = 19$ Substitute $y = 8$ and $z = 4$ into ①.

$x + 3(8) - 2(4) = 19$

$x = 3$

The solution of the system is (3, 8, 4).

Exercises

Solve each system by substitution. Check your answers.

7. $\begin{cases} -4x + 3y - 2z = 7 \\ 2x - 2y + 3z = 15 \\ -x + 2y - 2z = -6 \end{cases}$

(−2, 7, 11)

8. $\begin{cases} -2x - y + 5z = -5 \\ 3x + 2y - 7z = 10 \\ -2x - 3y + 6z = -12 \end{cases}$

(3, 4, 1)

9. $\begin{cases} 2x - y + 3z = 15 \\ 3x - 4y - 2z = 7 \\ 2x + 2y + 5z = 22 \end{cases}$

(5, 1, 2)

10. $\begin{cases} -2x + 2y - z = -1 \\ x + 2y + 2z = 11 \\ 4x - 3y + 2z = 4 \end{cases}$

(1, 2, 3)

11. $\begin{cases} x - 3y + 2z = 7 \\ 2x - 5y - 4z = -1 \\ x + 4y - 6z = -2 \end{cases}$

(6, 1, 2)

12. $\begin{cases} 3x - y - 3z = -13 \\ 2x + 4y - 5z = -5 \\ -5x + 2y + z = 3 \end{cases}$

(2, 4, 5)

Name ______________________ Class ____________ Date ____________

Reteaching

Solving Systems Using Matrices

Problem

How can you represent the system of equations with a matrix? $\begin{cases} 4x - 3y + 5z = -13 \\ x + 3y = 3 \\ -2x + 4y + 3z = 17 \end{cases}$

Step 1 Write each equation in the same variable order. Line up the like variables. Write in variables that have a coefficient of 0.

$$\begin{cases} 4x - 3y + 5z = -13 \\ x + 3y + 0z = 3 \\ -2x + 4y + 3z = 17 \end{cases}$$

Step 2 Write the matrix using the coefficients and constants. Remember to enter a 1 for variables with no numeric coefficient.

$$\left[\begin{array}{ccc|c} 4 & -3 & 5 & -13 \\ 1 & 3 & 0 & 3 \\ -2 & 4 & 3 & 17 \end{array}\right]$$

Exercises

Write a matrix to represent each system.

1. $\begin{cases} 2x + y = -3 \\ 3y = 5 \end{cases}$

$\left[\begin{array}{cc|c} 2 & 1 & -3 \\ 0 & 3 & 5 \end{array}\right]$

2. $\begin{cases} 3x - 5y + 2z = 9 \\ 4x + 7y + z = 3 \\ 2x - z = 12 \end{cases}$

$\left[\begin{array}{ccc|c} 3 & -5 & 2 & 9 \\ 4 & 7 & 1 & 3 \\ 2 & 0 & -1 & 12 \end{array}\right]$

3. $\begin{cases} 5x - y + 3z = 2 \\ 3y + 2z = 6 \\ 4x + 3y + z = 1 \end{cases}$

$\left[\begin{array}{ccc|c} 5 & -1 & 3 & 2 \\ 0 & 3 & 2 & 6 \\ 4 & 3 & 1 & 1 \end{array}\right]$

4. $\begin{cases} 2x - z = 3 \\ 5y + 4z = -5 \\ -x + 2y = 1 \end{cases}$

$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 3 \\ 0 & 5 & 4 & -5 \\ -1 & 2 & 0 & 1 \end{array}\right]$

5. $\begin{cases} z = 6 \\ x + y = 2 \\ 3x - 2y - 5z = 10 \end{cases}$

$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 6 \\ 1 & 1 & 0 & 2 \\ 3 & -2 & -5 & 10 \end{array}\right]$

6. $\begin{cases} 2z - 5x + 3y = 4 \\ -y + 2x + 4z = -2 \\ 3x + z - 2y = -5 \end{cases}$

$\left[\begin{array}{ccc|c} -5 & 3 & 2 & 4 \\ 2 & -1 & 4 & -2 \\ 3 & -2 & 1 & -5 \end{array}\right]$

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Reteaching (continued)

Solving Systems Using Matrices

Problem

What is the solution of the system? $\begin{cases} 2x + 5y = 5 \\ -x + 2y = -7 \end{cases}$

Step 1 Write the matrix for the system.

$$\left[\begin{array}{cc|c} 2 & 5 & 5 \\ -1 & 2 & -7 \end{array}\right]$$

Step 2 Multiply Row 2 by 2. Add to Row 1. Replace Row 1 with the sum. Write the new matrix.

$$\begin{array}{rrrr} & 2 & 5 & 5 \\ +2(& -1 & 2 & -7) \\ \hline & 0 & 9 & -9 \end{array} \qquad \left[\begin{array}{cc|c} 0 & 9 & -9 \\ -1 & 2 & -7 \end{array}\right]$$

Step 3 Divide Row 1 by 9. Write the new matrix.

$$\tfrac{1}{9}(0 \quad 9 \quad -9) \qquad \left[\begin{array}{cc|c} 0 & 1 & -1 \\ -1 & 2 & -7 \end{array}\right]$$

Step 4 Multiply Row 1 by −2. Add to Row 2. Replace Row 2 with the sum. Write the new matrix.

$$\begin{array}{rrrr} -2(& 0 & 1 & -1) \\ + & -1 & 2 & -7 \\ \hline & -1 & 0 & -5 \end{array} \qquad \left[\begin{array}{cc|c} 0 & 1 & -1 \\ -1 & 0 & -5 \end{array}\right]$$

Step 5 Multiply Row 2 by −1. Write the new matrix.

$$-1(-1 \quad 0 \quad -5) \qquad \left[\begin{array}{cc|c} 0 & 1 & -1 \\ 1 & 0 & 5 \end{array}\right]$$

This matrix is equivalent to the system $\begin{cases} y = -1 \\ x = 5 \end{cases}$. The solution is (5, −1).

Exercises

Solve each system of equations using a matrix.

7. $\begin{cases} 4x + 3y = 6 \\ -x - y = -1 \end{cases}$
(3, −2)

8. $\begin{cases} 6x + y = -2 \\ -x + 3y = 13 \end{cases}$
(−1, 4)

9. $\begin{cases} 3x + 2y = -4 \\ -4x - 3y = 7 \end{cases}$
(2, −5)